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Optimization of repetition spacing in the practice of learning

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Short communication

Abstract. A universal formula for computing inter-repetition intervals in paired-associate learning has been determined for the knowledge retention level of 95%. It is claimed that the formula could be used in the practice of learning for a wide range of subjects, regardless individual learner's capacity.

Key words: memory, learning, paired-associate learning, spacing repetitions

The impact of different inter-repetition intervals on the effectiveness of learning has been widely discussed in a number of publications (Glenberg and Lehman 1980, Bahrick and Phelps 1987). In particular, the spacing effect has been discovered as a universal phenomenon (Hintzman 1974, Glenberg 1977). Spacing effect consists in better performance in learning tasks if the spacing of repetitions is distributed, i.e. sparse, as opposed to massed spacing. Consequently, it has been proposed that the optimum inter-repetition intervals used in learning are the longest intervals that do not result in forgetting (Bahrick and Phelps 1987). Forgetting however, has a stochastic nature and it is impossible to predict when it will occur in a particular case. Therefore, despite numerous attempts, there have been very few reports postulating repetition spacing that could be used in the practice of learning (Atkinson 1972, Bahrick and Phelps 1987).

In the presented study a stochastic approach has been assumed. The optimum spacing of repetition has been computed by defining an optimum interval as the interval which causes a small, previously determined fraction of the learned material to be forgotten. A computer program has been applied to supervise the learning process in such a way that to make sure that only 5% of to-be-remembered items are not remembered at the moment of repetition. The program employed an optimization algorithm to lengthen or shorten inter-repetition intervals in case the proportion of forgotten items dropped below or increased above the desired level of 5%. Moreover, in the learning process, to-be-remembered items have automatically been divided into difficulty categories, depending on the subjects' performance. For each of the difficulty categories, a different repetition spacing was applied. The subjects taking part in the experiment were 7 unpaid volunteers, students of computer science at the Technical University of Poznań. In the period of 18 months, they memorized and repeated altogether over 35000 items of their choice (the items had the form of Polish-English word pairs). The memorized items were had not been known to the subjects before the experiment. Subjects mastered the entire material in equal portions over the period of 2 months using the algorithm described below and continued repetitions over the remaining 16 months.

Each repetition in the algorithm had the following course:

1. presenting the question
2. subject's attempt to respond
3. comparing the response with the correct answer
4. self-assessment in a 0-5 grade point scale (0 - very bad, 5 excellent)

The to-be-found function determining the optimum intervals between repetitions, later called the function of optimum intervals, was represented in a tabular form as a matrix OF, for Optimal Factor (Woźniak and Biedalak 1992):

$$I(EF,1)=OF(EF,1) \qquad (1)$$

$$I(EF,R)=I(EF,R-1)*OF(EF,R) \qquad (2)$$

where:
EF - easiness factor which was intended to reflect the easiness with which the item is remembered.
I(EF,R) - interval, expressed in days, before the R-th repetition for items whose easiness factor was determined to be EF.

An optimization algorithm was used to modify the initial value of the matrix OF in order to get a better approximation of the function of optimal intervals, the main criterion being a stable knowledge retention of 95%.

Two different classes of OF matrices were used:

- univalent, with all entries initialized at the same value: for all EF and for all R, OF(EF,R):=1.5.
- predetermined with variable OF entries for different difficulty categories (Woźniak and Biedalak 1992): for all EF, OF(EF,1):=5, and for all EF and R, OF(EF,R):=EF; where EF was chosen to equal 1.3, 1.4, 1.5, ..., 3.2 (see Table I).

The predetermined matrix OF was intended to ensure faster convergence of OF entries to their optimum value while the univalent matrix OF was

TABLE I

Predetermined matrix of optimal factors (EF - easiness factor, R - repetition number)

Optimal factors (OFs) before the experiment

R\EF	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5
1	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
2	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00	2.10	2.20	2.30	2.40	2.50
3	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00	2.10	2.20	2.30	2.40	2.50
4	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00	2.10	2.20	2.30	2.40	2.50

used to verify the validity of the optimization algorithm. 15% of items were subject to optimization based on the univalent OF matrix, and 85% of items were scheduled using the predetermined OF matrix.

The following learning algorithm was used (note that repetitions may proceed indefinitely and the experimental value of the OF matrix may be obtained at any time, the later the better):

1. Memorize a new item i (i.e. learn the association between the question and the answer in a self-paced manner)
2. EFi:=2.5 (set the initial value of the EF factor corresponding to the item i)
3. Ri:=1 (set the repetition number to one)
4. Use the matrix OF to determine the date of the first repetition Eqn. (1)
5. On the day the repetition was scheduled repeat the item by answering the relevant question and assess the quality of the response in a 0-5 grade scale (high grades for good performance)
6. If the learner wishes to continue learning then Ri:=Ri+1 else STOP
7. Decrease the value EFi in case of quality lower than 4, increase it otherwise. Items that cause problems in learning will be classified as more difficult and consequently subject to different repetition spacing (see Woźniak and Biedalak 1992 for more details)
8. Decrease the OF(EFi,Ri - 1) value in case its application yielded quality lower than four, increase it otherwise. Consequently, unsuccessful repetitions will shorten the intervals used in learning (the pace of changes was adjusted to obtain a stable retention of 95% at repetitions; drop in retention would favor slower upward and faster downward changes of OF values thus reducing the length of intervals used in repetitions; (see Woźniak and Biedalak 1992 for more details)
9. If the quality was greater than or equal to three then schedule the next repetition according to Eqn. (2) and go to Step 5. If the quality was lower than three then go to Step 3 (the item is considered forgotten).

An exemplary OF matrix produced in the course of the experiment is presented in Table II (this matrix was derived from the univalent OF matrix).

Upon completing the data-collecting process, the results have been processed by using a range of approximation procedures designed to obtain the best-fitting cumulative function of optimal intervals that could practically be used in classroom learning procedures (Rosenbrock 1960).

The universal formula describing the function of optimum inter-repetition intervals, for the knowledge retention of 95%, has been determined as follows:

$$OI(EF,1)=5 \qquad (3)$$

$$OI(EF,R)=OI(EF,R-1)*(EF-0.1+e^{-2.3*R+5}) \qquad (4)$$

where:

EF - easiness factor characteristic for a given to-be-remembered item (usually between 1.3, for the most difficult, and 2.8, for the easiest items)

R - number of the repetition

TABLE II

An exemplary matrix of optimal factors obtained in the course of the experiment. Note, that a smoothing algorithm was used in order to establish the value of the entries that could not be computed because of a too short experimental period (EF - easiness factor, R - repetition number)

	Optimal factors (OFs) after the experiment												
R\EF	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5
1	5.56	11.3	12.7	12.7	11.6	9.27	7.68	6.64	6.93	7.30	10.9	10.8	9.88
2	1.54	2.07	2.27	2.46	3.24	2.71	2.77	3.10	3.30	3.77	4.16	4.19	4.05
3	1.20	1.27	1.14	1.35	1.41	1.56	1.77	1.97	2.27	2.22	2.09	2.19	2.30
4	1.22	1.25	1.26	1.41	1.47	1.52	1.65	1.76	1.81	1.87	1.88	2.04	2.17
5	1.22	1.30	1.28	1.32	1.43	1.49	1.60	1.60	1.65	1.69	1.67	1.83	2.01
6	1.24	1.27	1.28	1.33	1.39	1.48	1.69	1.79	1.82	2.33	2.25	2.33	2.23
7	1.27	1.27	1.28	1.34	1.41	1.56	1.68	1.97	1.99	1.98	2.18	2.11	2.09
8	1.28	1.29	1.31	1.34	1.41	1.53	1.65	1.90	1.92	1.95	1.98	1.98	2.00
9	1.32	1.34	1.38	1.44	1.57	1.56	1.66	1.95	1.83	1.85	1.88	1.90	1.95
10	1.35	1.41	1.46	1.53	1.64	1.61	1.67	1.79	1.71	1.76	1.80	1.85	1.91
11	1.40	1.38	1.39	1.43	1.51	1.67	1.57	1.62	1.65	1.70	1.76	1.82	1.89
12	1.32	1.36	1.33	1.36	1.41	1.45	1.50	1.55	1.61	1.67	1.74	1.81	1.88
13	1.23	1.26	1.28	1.31	1.35	1.40	1.46	1.52	1.59	1.66	1.73	1.81	1.88
14	1.20	1.22	1.24	1.27	1.31	1.37	1.44	1.51	1.58	1.66	1.73	1.81	1.88
15	1.20	1.20	1.21	1.25	1.29	1.36	1.43	1.51	1.58	1.66	1.73	1.81	1.88
16	1.20	1.20	1.21	1.23	1.29	1.36	1.43	1.51	1.58	1.66	1.73	1.81	1.88
17	1.20	1.20	1.20	1.23	1.29	1.36	1.43	1.51	1.58	1.66	1.73	1.81	1.88
18	1.20	1.20	1.20	1.23	1.29	1.36	1.43	1.51	1.58	1.66	1.73	1.81	1.88
19	1.20	1.20	1.20	1.23	1.29	1.36	1.43	1.51	1.58	1.66	1.73	1.81	1.88
20	1.20	1.20	1.20	1.23	1.29	1.36	1.43	1.51	1.58	1.66	1.73	1.81	1.88

OI(EF,R) - optimal interval for items of difficulty EF before the R-th repetition (in days)

Statistical differences between the parameters of the functions presented in Eqns. (3) and (4) for different subject have been found insignificant. Similarly, the difference between matrices derived from the univalent and predetermined values were insignificant. The obtained results unequivocally indicate the superiority of progressive repetition spacing over massed or distributed spacing and may be used in the practice of learning in order to minimize time necessary for memorization and retention of the learned material.

Software used in the reported research is available from the authors free of charge upon sending a 3.5 in, 1.44 MB diskette.

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