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Summation of two illusions of extent

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In the present visual psychophysical study, the Oppel-Kundt and and Müller-Lyer illusion magnitudes were measured separately (by single figures) and in combination (by two patterns superposed spatially). Data for 30 subjects revealed extensive variability both for the separate and combined illusion strength. Nevertheless, the effect of addition of the perceived length distortions was established. The combined illusions were significantly stronger than the separate ones. Dynamics of the misperceptions summation was studied by varying length of the Müller-Lyer wings in the superposed stimuli. According to the experimental data obtained, the two misperceptions in length occurred and combined into sensory response varying in dependence on the spatial parameters of the superposed stimuli and on the individual experimental accomplishment. The data supported an explanation for the origin of the filled/unfilled illusion: overestimate of a filled interval length developed due to the spatiotemporal integration along a continuous excitation path elicited by the real or imaginary contours of the filling.

Key words: geometrical illusions, Oppel-Kundt, Müller-Lyer, stimuli superposition

INTRODUCTION

Experimental data on the summation of perceived length distortions have been obtained in examinations of the Müller–Lyer (M-L) and Oppel–Kundt (O-K) stimuli with varying orientations of the reference and test parts in the visual field (Bertulis and Bulatov 2005, Bulatov and Bertulis 1999). These findings provided the pure characteristics of visual field anisotropy and geometric illusions and showed the relationships between them. The simultaneous appearance of anisotropy with either illusion was combined in an algebraic summation.

A sign of the algebraic summation of the perceived extent distortions having opposite signs could be discerned in the experiments with the M-L and O-K figures combined together in a way that the inward facing M-L wings overlapped the O-K filling stripes within the stimulus reference interval, and the outward wings flanked the empty test interval (Bulatov and Bertulis 2005). No illusion occurred if the superposing figures were either dark or light, and one of the figures was as dark as the other was light. Evidently, underestimates of the interval length due to the inward wings (the M-L effect) counterbalanced the overestimate of the same interval extent because the filling stripes (the O-K illusion). The resulting distortion approached zero. However, the illusion appeared when different contrasts of different figures were used. In other words, one of the opposite signals became outbalanced and perceivable.

In contrast, the absence of additivity of misperceptions was reported in the superposed O-K figures study (Bertulis et al. 2014). Three basic O-K patterns having either a regular sequence of uniform stripes or a contour rectangle, or a solid filling in the reference stimulus interval produced illusory distortions about equal in magnitude. Two types of the combined stimuli: the stripes within the rectangle and the stripes on the smooth block showed the same values that neither increased nor decreased the level.

We have assumed that the latter literature data did not contradict the previous findings. Presumably, during the length matching procedures, the strength of the resulting perceptual distortions of extent might be determined by the combined errors of different

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neuronal mechanisms converging toward the "decision-making" input. Each of the free-standing mechanisms might strengthen or weaken the final perceptual error to a certain degree in dependence on the parameters of the stimuli combined. But, when the superposed stimuli were processed in a single neural network, no additional component could occur to vary the output signal.

Indeed, the three types of perceived length distortions (anisotropy of the visual field, the M-L illusion, and the O-K illusion) may differ in their origin. Orientation anisotropy is explained by means of the "framing effect" of the elliptical shape of the visual field (Kunnapas 1957) and by low-level neuronal models, such as neural sensitivity (Rose and Blakemore 1974), neuronal tuning (Andrews 1967, Thomas and Gille 1979), and neuronal density (Mansfield 1974, Orban and Kennedy 1981, Rose and Blakemore 1974), or by non-homogeneity of the magnification factor on the striate cortex (Bulatov et al. 1996). The M-L illusion is basically related to a perceived positional shift of the gravity center of the excitation profile formed by stimulus terminals and distracters (Bulatov et al. 2009, Morgan et al. 1990). The O-K illusion appears to arise due to its own specific reasons also, e.g., spatiotemporal integration along a continuous excitation path elicited by real or imaginary contours of the filled space (Bertulis et al. 2014).

Consequently, the summation of distortions was observed in the length matching task experiments with single M-L and O-K stimuli of variable orientations and maybe at their superposition but not in the experiments with a combination of the stimuli of the same O-K category.

The aim of the present study was to establish addition of distortions of perceived extent and to show experimentally that the M-L and O-K events might arise simultaneously and combine together during the length estimation procedure. The task was to measure quantitatively the effect of combination of misperceptions of the same sign: i/ overestimate of the length of the stimulus interval flanked by arrowheads facing outward (M-L) and ii/ overestimate of the interval comprising smooth or regular filling (O-K), when compared with underestimate of an empty stimulus interval having inward M-L wings. Therefore, some modified M-L and O-K figures formed of spots or line segments were used in the experiments. In the superposed stimuli, none of the illusion inducers (shafts, regular fillings, outward and inward wings) overlapped or crossed both in the reference and test parts of the stimuli. By the method of adjustments, manifestation of separate and combined illusions of extent was examined in the first set of experiments (Experiment 1). Dynamics of summation was studied by varying length of the M-L wings of the superposed stimuli in the second set of experiments (Experiment 2).

METHODS

Stimuli and apparatus

Three spatial interval stimuli (Fig. 1) were used in the experiments, as possessing left-right symmetry and producing stronger illusions; the three-part O-K figure with two filled intervals flanking the empty part induced a 25% stronger illusion than the two-part figure with one filled interval (Bertulis et al. 2009).

The control stimulus was comprised of four spots (Fig. 1*A1*) serving as terminals for the three unfilled spa-

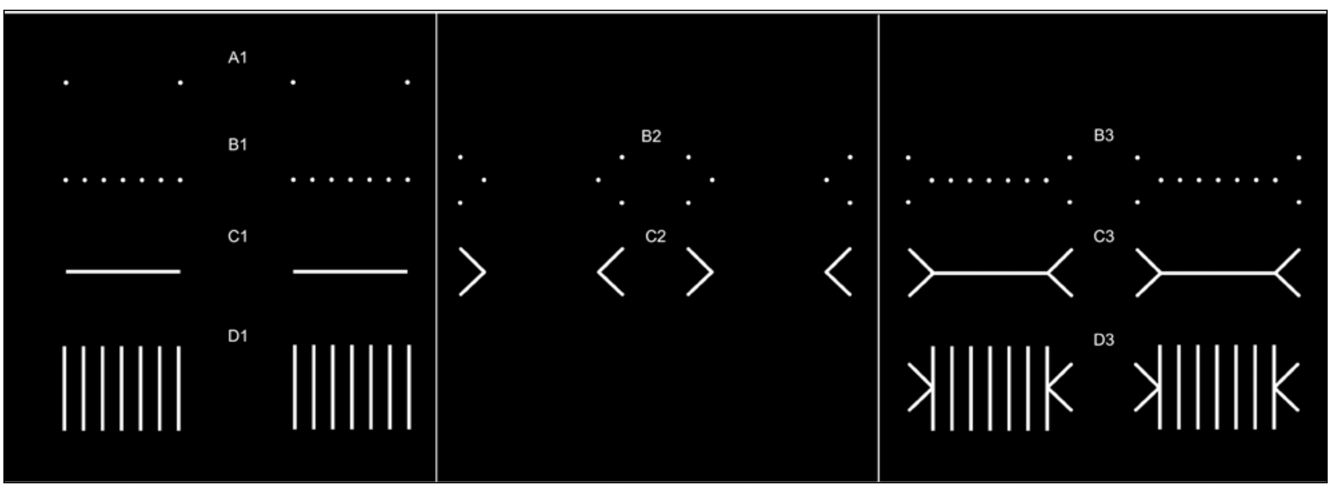


Fig. 1. Facsimiles of the three-part stimuli of the Oppel-Kundt and Müller-Lyer type used in the experiments. *A*, the control stimulus formed of four terminal spots; *B1*, *C1*, and *D1*, the Oppel-Kundt stimuli with spots, horizontal line segments, or vertical stripes in the referential intervals, respectively; *B2* and *C2*, the Müller-Lyer figures formed of spots or line segments; *B3*, *C3*, and *D3*, the superposed stimuli. See main text for a detailed explanation.

tial intervals: two laterals, the references, 90 arc min for each in extension and one medial, the test, which varied in length.

For the O-K stimulus, two filled intervals (90 arc min in length) flanked the medial empty space, the length of which was varied by subjects in the experiments. The regular sequences of seven spots (Fig. 1*B1*), horizontal line segments (Fig. 1*C1*), or seven vertical stripes (66 arc min high; Fig. 1*D1*) served as fillings. The filled and unfilled intervals in the stimuli were the reference and test parts, respectively. When the testing interval length was changed, the reference flanks moved symmetrically outside or inside without any structural or metric changes.

In the M-L figures (Fig. 1*B2* and Fig. 1*C2*), four pairs of wings designated the three intervals. Two lateral pairs were facing outward and two medial pairs were directed inward. The wings were formed of spots or line segments. Wing length was either 20 arc min (in Experiment 1), or it varied from 0 to 34 arc min and was considered an independent variable (in Experiment 2). The wing opening angle was fixed at 90°. The distance between the apexes of the wings was 90 arc min in the lateral (reference) stimulus intervals, but it varied in the medial (testing) part, as in the O-K stimulus. No shaft lines were present in the M-L figures.

The spot diameter and line width in all stimuli was 2.2 arc min. The spot and line luminance was 55 cd/m^2. In the superposed stimuli, the M-L and O-K figures coincided precisely with the reference terminal and the testing intervals matched. No superposition of spots or lines and no luminance summation occurred.

All stimuli were drawn by the Cambridge Research Systems VSG 2/3 and displayed on an EIZO T562 monitor calibrated and gamma corrected using a Cambridge Research Systems OptiCAL photometer. The Psychophysical Experiment Toolbox in Mathworks Matlab software platform controlled the presentations of the stimuli, introduced changes according to the subject's command, and recorded the subject's responses.

The subjects observed the monitor screen monocularly through a 3 mm diameter artificial pupil to minimize optical aberrations. A chin holder was used to limit head movement. The distance between the monitor screen and the subject's eye was 260 cm, which caused the screen pixels to subtend 0.55 × 0.55 arc min. The stimuli were presented horizontally against a 0.1 cd/m^2 luminance background.

Subjects

Twenty-nine University students (eleven male and eighteen female; mean age 20.7 years; SD=1.8) took part in Experiment 1 and twenty-eight of them participated in Experiment 2. None of the students had practiced any similar experiments before and were naive with respect to the goals of the investigation. One of the manuscript's authors, a university lecturer participated in the present experiments and completed the program. The lecturer's data were taken into consideration together with those of the students. All subjects were normally sighted or were wearing their usual optical corrections.

Procedure

The experiments were carried out in a dark room. The method of adjustment was used to establish functional dependence of illusion strength on the spatial parameters of the superposed or separately presented stimuli. Biases in the judgment criteria, which are an inherent characteristic of the method, were reduced by randomizing stimuli with different parameters (including the control) in the presentation sequences.

Subjects were asked to vary the test interval length by a pixel at a time (by manipulating the keyboard buttons) and to establish equal lengths for the three stimulus intervals. The initial length of the test interval was randomized and distributed evenly within a range of 90 ± 15 arc min. The subjects did not know in advance whether the computer program would present the test interval longer or shorter than the reference interval (90 arc min). The superposed and separate stimulus presentations were randomized during the experimental sessions. No instructions concerning gaze fixation were given, and observation time was effectively unlimited. The subjects adjusted the test interval somewhat longer than the reference flanks while performing the experimental task, and the errors made (error, arc min=adjusted length, arc min – 90 arc min) were considered the illusion strength values. The subjects carried out four experimental sessions on different days with five trials for each stimulus, i.e., 20 trials were included for each data point analysis. Ten trials were performed for the varying wing length experiments.

Before the experimental sessions, the students were trained to match the three spatial intervals of the control and proper stimuli on 2–3 different days. The students were not given any additional practice or knowledge of the results during the experimental sessions.

The illusion summation effect was studied using three versions of the superposed stimuli, such as B3, C3, and D3 (Fig. 1) in which M-L wing length was fixed at 20 arc min (Experiment 1). Summation dynamics was examined by the varying length of the M-L wings 0–34 arc min in the superposed stimuli (Experiment 2).

Analysis

The Matlab Mathsoft and SigmaPlot Systat Software were used for data analysis. The Kolmogorov-Smirnov with Lilliefors corrections distribution normality test was applied to data gathered at eight different conditions in the experiments. For all cases, when the normality of frequency distribution was not rejected, the parametric methods were used. A one-way repeated measures ANOVA was used to analyse the data followed by a two-tailed t-test Holm-Sidak *post hoc* test for multiple pairwise comparisons.

RESULTS

A length averaging procedure was performed when testing the three-part stimuli instead of symmetry processing or bisection, which are usually used for two-part tests. In the present experiments, the subjects reported no difficulties performing the length matching task when manipulating the three-part stimuli and were able to judge the required sizes by varying the test interval of both the single and superposed stimuli. One-way repeated measures ANOVA showed a significant overall variation ($F=16.04$, $P<0.05$) of illusion strength for eight different stimuli.

Length matching control

The control three-part stimuli had no distracters or inducers (Fig. 1*A1*) but caused length judgment biases like the two-part stimuli in the bisection procedure (Bulatov et al. 1997). The average bias was -3.3 arc min (SD=4.6). Five subjects judged the middle interval as being longer than the two laterals and, therefore, made it shorter while trying to achieve perceived equality. The biases of the subjects' perception were considered a positive sign (Table I). Twenty-five subjects perceived the test as shorter and produced it longer than the references; their errors were considered negative. None of the subjects was precise enough to show zero deviation. Fourteen subjects gave relatively small deflections within an interval from 1 to 3 arc min (the absolute values of the errors). Seven subjects made 4–6 arc min errors and nine made 7–10 arc min errors (Table I).

The control data were used to correct illusion strength obtained in the experiments: the positive errors were added to the measured illusion values of the subjects, and the negative ones were subtracted. The 3 arc min criterion was considered a conventional change step in the M-L and O-K illusions magnitude and in their common manifestation values.

Experiment 1

Single illusions

Both the O-K and M-L illusions varied greatly in strength from person-to-person. The individual values of the M-L illusion were arranged in increasing rank order (Fig. 2*A* and 2*B*) and illustrated a rather wide scatter. However, the two stimulus versions, such as the M-L wings formed of lines and those of spots, elicited similar perceptual errors, such as 2.2 arc min and 3.6 arc min at minimum and 34.6 arc min and 38.5 arc min at maximum.

The frequencies of the variants taken according to the value change step of 5.4 arc min (Fig. 2*Aa* and 2*Bb*), were irregular and asymmetric in shape because the sample was relatively small (n=30). However, the Lilliefors normality test ($P=0.172$ for spots and $P=0.923$ for lines) revealed a certain resemblance to the normal distribution.

The two rankings in Fig. 2 corresponded to different observer orders. Nevertheless, 21 observers (1, 2, 3, 5, 7, 8, 9, 12, 13, 14, 17, 19, 26, 20, 21, 22, 23, 24, 25, 28, and 29) maintained rather close error values (difference not exceeding 3 arc min, one conventional step) for the two stimuli. Four subjects (4, 15, 27, and 30) experienced larger differences of 4–6 arc min (two steps), and five (6, 10, 11, 16, and 18) had still larger divergence of 10–18 arc min for illusions' strength.

Three different O-K stimulus versions yielded results (Fig. 3) similar to previous data (Fig. 2), illustrating a typ-

Table I. Biases of the perceived length equality in the control stimulus.

Error, arc min	Subjects
8	7 10
3	1 6
1	11
0	
-1	3 18
-2	4 12 20 23
-3	2 8 15 21 29
-4	14 16
-6	5 13 24 25 30
-7	26 19
-8	9 28
-9	27
-10	17 22
	Total: 30

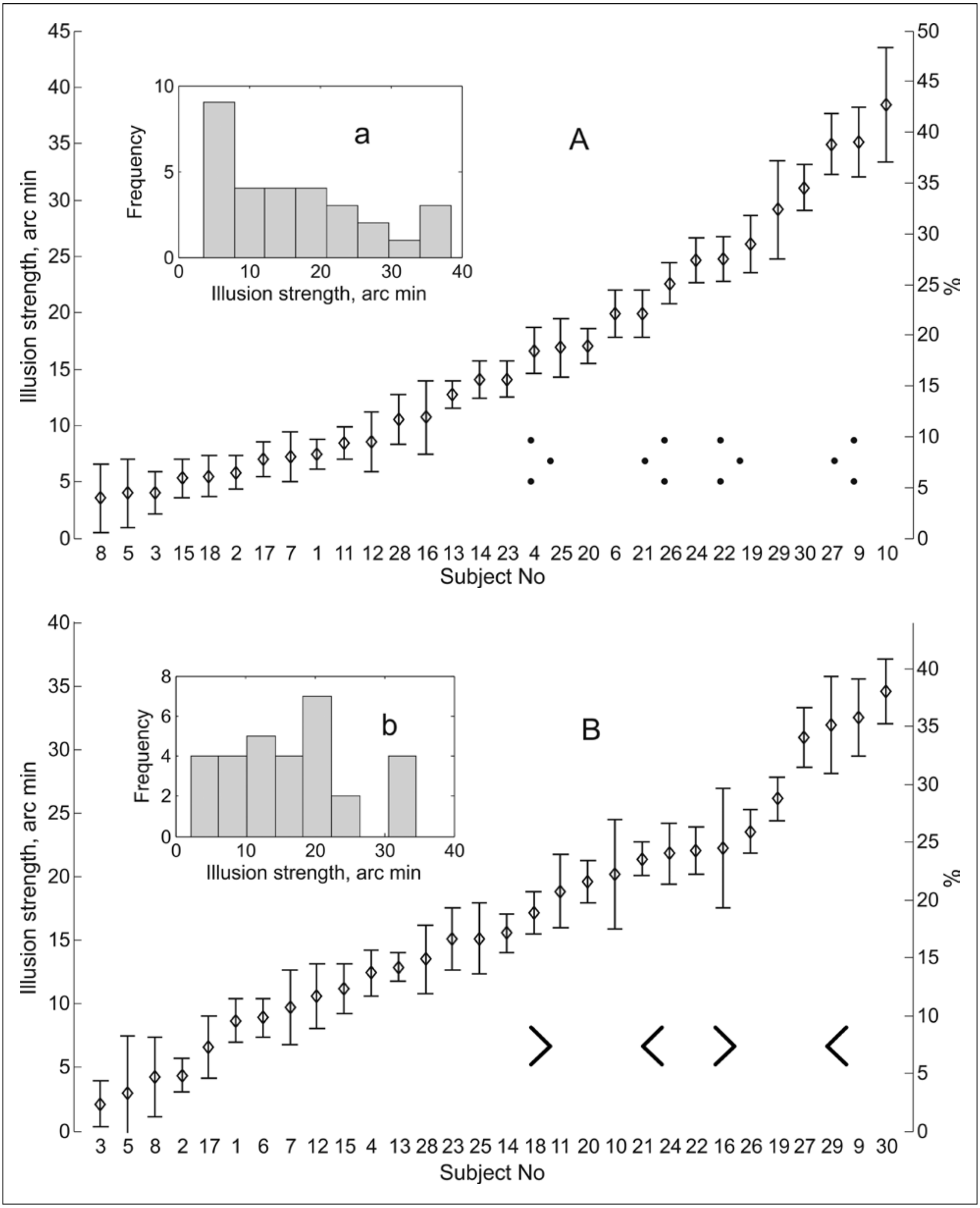
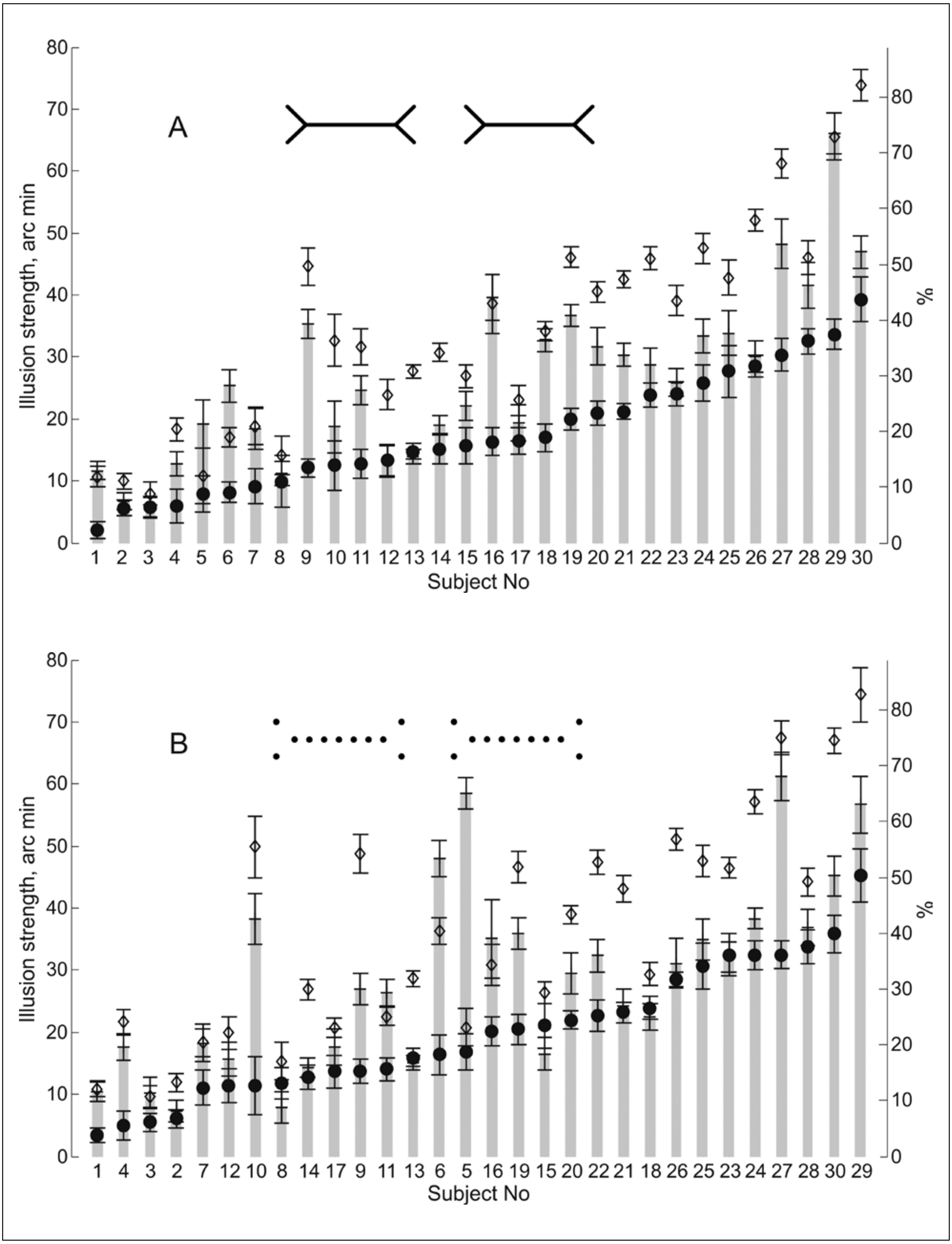


Fig. 2. Rank ordering of the Müller-Lyer illusion strength (the length matching error) for the stimuli of spots (in *A*) and of lines (in *B*). Data for 30 subjects (horizontal line numbers). In *a* and *b*, the strength value frequencies in the sample classes gradated according to the value change step, 5.4 arc min. Error bars, 0.95 confidence intervals.


A
Illusion strength, arc min
Subject No
%
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
B
Illusion strength, arc min
Subject No
%
1 4 3 2 7 12 10 8 14 17 9 11 13 6 5 16 19 15 20 22 21 18 26 25 23 24 27 28 30 29

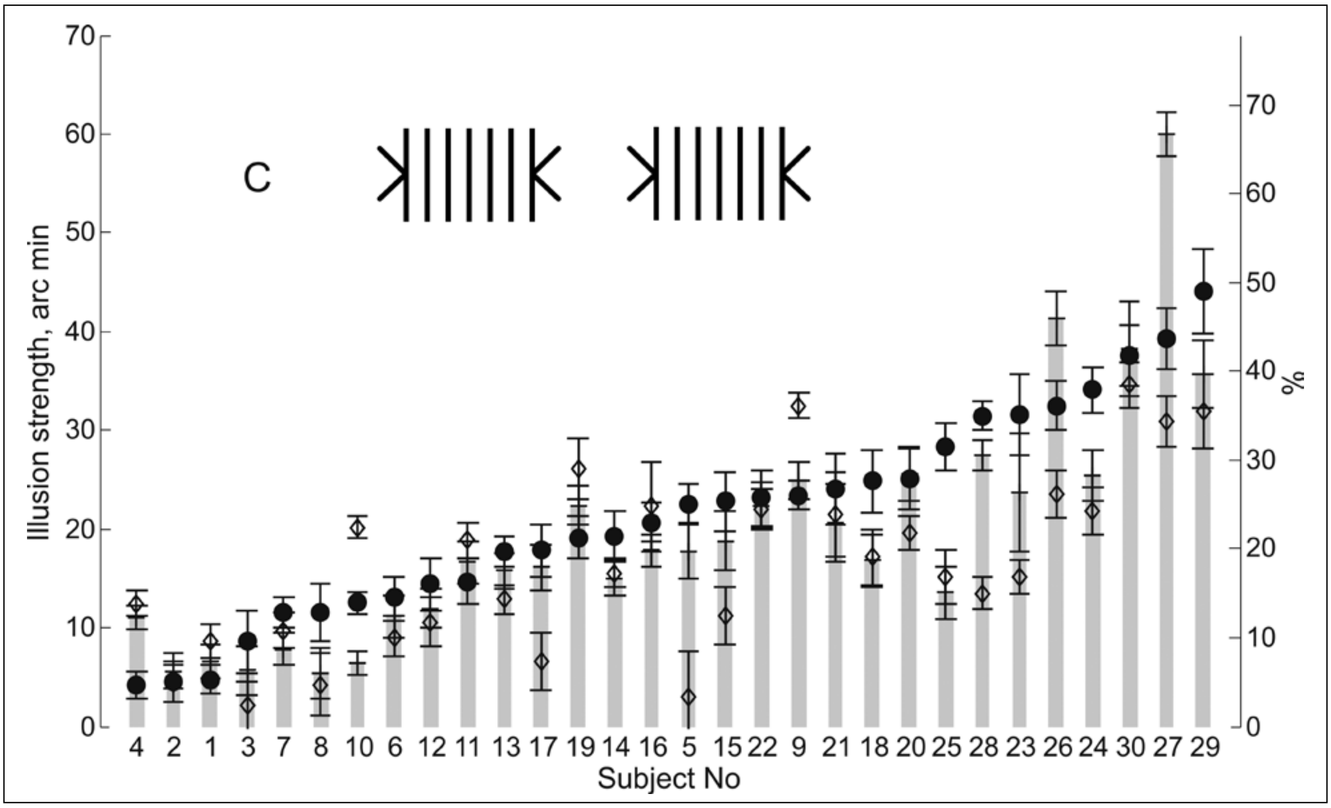


Fig. 3. Psychophysical result of the Oppel-Kundt and Müller-Lyer stimuli superposition, gray vertical bars. Dark circles, the Oppel-Kundt illusion strength for the stimuli with the filling lines (in *A*), with spots (in *B*), with vertical stripes (in *C*). Diamonds, the Müller-Lyer (M-L) illusion added to the Oppel-Kundt illusion to show the physical sum of two distortions of extent (Fig. 3A and Fig. 3B). In Fig. 3C, the M-L illusion values fit the figures on the y-axis. Error bars, 0.95 confidence intervals. Facsimiles of the superposed stimuli are seen above each data set. Data for 30 subjects.

ically wide dispersion in the length matching errors. The lowest values on the O-K illusion appeared at 2 arc min for line filling (Fig. 3*A*), 3.4 arc min for the spots (Fig. 3*B*), and 4 arc min for the vertical stripes (Fig. 3*C*). The highest values were at 39.3 arc min for the lines (Fig. 3*A*), 45.2 arc min for the spots (Fig. 3*B*), and 44.1 arc min for the vertical stripes (Fig. 3*C*). The frequencies of the variants taken according to the value change step of 5.4 arc min (Fig. 4) were irregularly shaped because they were the same subject sample (n=30). The similarity of the frequencies to a normal distribution was supported by the Lilliefors test (P=0.402 for lines, P=0.613 for spots, and P=0.975 for stripes).

The three O-K value rank orderings corresponded to different subject ordering (Fig. 3) as with the M-L data (Fig. 2). The subjects' judgments were affected by the stimulus structure, but to different extents. For example, subject 11 showed similar illusion values (12.5, 13.6, and 15.3 arc min) tightened within a 3 arc min interval (one conventional step), whereas the values for subject 5 were more broadly distributed (7.9, 16.8, and 22.5 arc min) within the 14.6 arc min interval (almost five conventional steps). The mean value of the M-L illusion (Fig. 5) caused by the lines was 16.6 arc min (SD=9.0) and that evoked by spots was 16.2 arc min (SD=10.4, paired t-test, t_{29}=0.21, P>0.05). In average, the M-L illusion magnitude was 18% of the reference length.

The mean values for the O-K illusion were 17.6, 19.8, and 21.3 arc min (SD=9.5, 10.5 and 10.4) considering the lines, spots, and vertical stripes, respectively (Fig. 5). In other words, the O-K illusion for the spot stimulus was greater by only 2.2 arc min than that for the continuous line stimulus (paired t-test, t_{29}=1.3, P>0.05). But this was in agreement with the basic property of the illusion: stimuli with the optimal number of filling elements were more effective than those with a solid filing (Ebbinghaus 1902, Obonai 1933, Piaget and Osterrieth 1953). The effect was interpreted in terms of the local integration processes generating additional repulsion between ultimate and penultimate filling elements (Mikellidou 2012).

The vertical stripes illusion was greater by only 3.7 arc min; (t_{29}=2.3, P>0.05; Fig. 5) than that of the horizontal lines. The tendency might occur due to perceptual properties of excitation continuity (Bertulis et al. 2014). The regular stripes in the stimulus formed two horizontal illusory contours, which, like the real ones, produced continuous excitation paths in the visual network, and the spatiotemporal integration along the paths may give somewhat larger extension values than those arising from the stimulus with a single line in the reference.

On average, the O-K illusion magnitude was 22% of the reference length.

The M-L illusion strength did not change (16.6 and 16.2 arc min) when the spot stimulus turned to that of line segments (Fig. 2 and Fig. 5). One might speculate that during the early stimulus processing stages, the low-level filtering highlighted the endpoints of the line segments of the M-L wings and made the excitation pattern similar to that of the spots. Therefore, positional shifts in the gravity centers might become about equal.

This was not the case with the three O-K figures. Their excitation patterns hardly became alike due to low-level spatial-frequency filtering. Nevertheless, the three versions of the O-K stimuli yielded perceived distortions similar in strength under present experimental conditions.

Moreover, the O-K and M-L illusions were approximately equal in strength. The stimuli formed of lines produced distortions 17.6 arc min and 16.6 arc min (t_{29}=0.63, P>0.05). The stimuli formed of spots induced 19.8 arc min vs.16.2 arc min (t_{29}=2.2, P>0.05) illusions. The O-K illusion given by the vertical stripes was 21.3 arc min, and the M-L illusion formed of lines was 16.6 arc min (t_{29}=2.9, P>0.05). The two illusions did not differ according to their manifestation manner. The mean of the standard deviations (Table II*A*) and the standard deviation of the means (Table II*B*) within each data set were about the same for the two distortions of perceived length.

Compound illusions

In the experiments with the line or spot stimuli (Fig. 3*A* and Fig. 3*B*), the superposed M-L and O-K figures caused greater distortions in perceived length (I_S) than single figures were able to produce ($I_{O\text{-}K}$) or ($I_{M\text{-}L}$). The increase of the compound illusion magnitude was considered a result of the summation of two distortions: a) overestimate of the interval length due to the outward wings and b) overestimate the length due to the inside filings with coefficient r_S, which is determined as the ratio $I_S/(I_{O\text{-}K}+I_{M\text{-}L})$, which can vary from 0 to 1. The complete and incomplete summation concept was used to represent the results quantitatively. Complete summation is an illusion approximately equal in strength to the arithmetical sum of two separate illusions ($I_S \approx I_{O\text{-}K}+I_{M\text{-}L}$ and r_S=1). An incomplete summation considers parity loss making the combined illusion brownout than the sum of its parts ($I_S<I_{O\text{-}K}+I_{M\text{-}L}$ and $0.5<r_S<1$). Finally, if the combined illusion appeared equal or less than the separate distortions, the absence

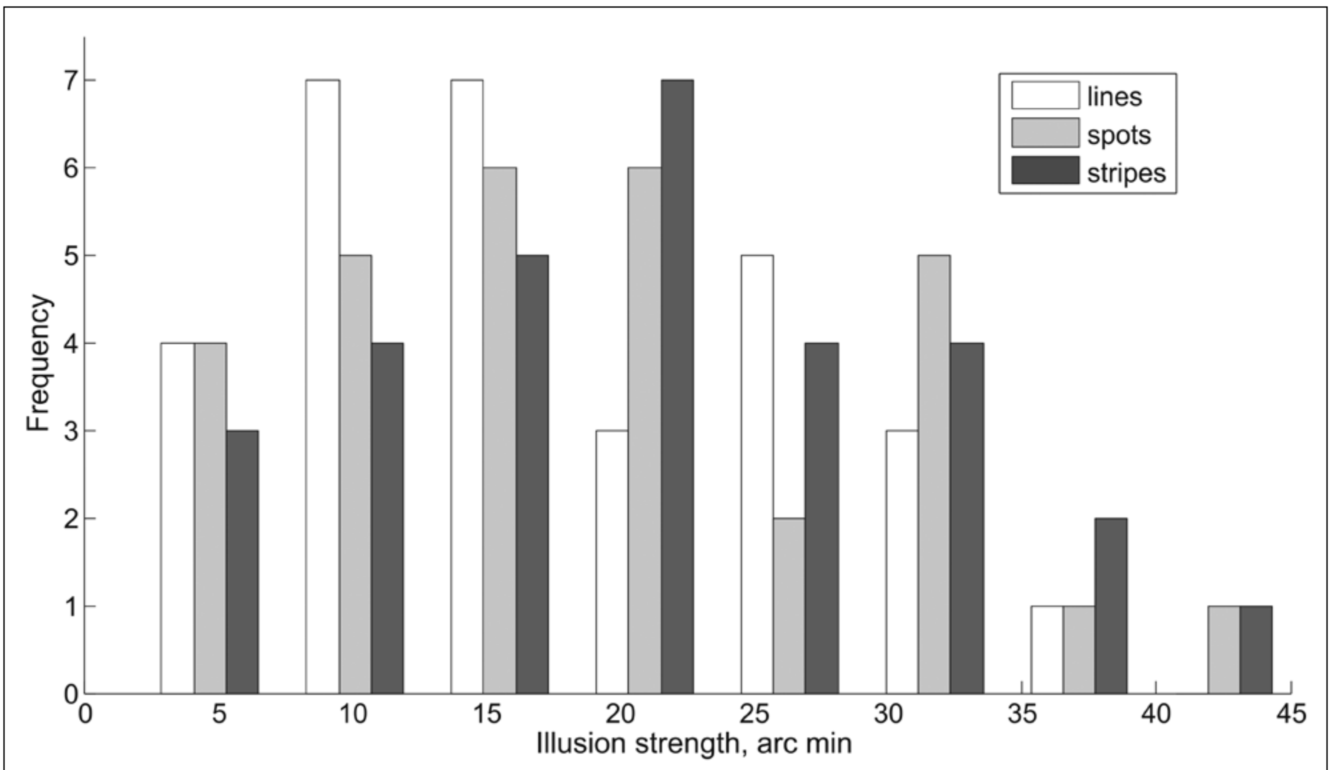


Fig. 4. Strength value frequencies in the Oppel-Kundt sample classes gradated according to the value change step, 5.4 arc min. White, gray, and black bars, the stimuli with lines, spots, or vertical stripes in the references, respectively. Data for 30 subjects.

Table II. The Oppel–Kundt and Müller–Lyer compound illusion strength, arithmetical sum (arc min), and summation coefficient r_S for 16 subjects in experiment I with stimulus D3 (Fig. 1).

	10	25	8	7	18	16	14	9	21	24	29	12	1	22	19	11
I_{O-K}	12.5	28.3	11.6	11.6	24.9	20.7	19.3	23.4	24.1	34.1	44.1	14.4	4.8	23.2	19.1	14.7
I_{M-L}	20.2	15.2	4.3	9.7	17.2	22.2	15.6	32.5	21.4	21.8	31.9	10.6	8.7	22.1	26.1	18.9
I_S	6.4	13.5	5.4	7.8	16.8	17.8	15.0	24.9	20.6	25.4	35.7	11.9	6.6	22.3	22.4	16.6
$I_{O-K} + I_{M-L}$	32.7	43.4	15.9	21.3	42.0	43.0	34.9	55.9	45.5	55.9	76.0	25.0	13.5	45.2	45.2	33.6
r	0.20	0.31	0.34	0.37	0.40	0.42	0.43	0.45	0.45	0.45	0.47	0.48	0.49	0.49	0.49	0.50

of summation was described as coefficient $r_S \leq 0.5$. An incomplete summation or summation absence could be caused by weakening of the responses to the stimulus components depending on the subject's visual processing properties.

The compound illusion strength (I_S) for the superposed line stimuli (Fig. 3*A*) varied from person to person between 5% and 74% of the reference length, whereas the strength limits for the separate line stimuli were 2–38% (I_{M-L}; Fig. 2B) and 2–44% (I_{O-K}; Fig. 3A). Complete summation (with deflection not exceeding ±3 arc min, one conventional step) was represented in the five observers' data (1, 7, 16, 18, and 29), and coefficient r_S approached 1. Seventeen cases of incomplete summation occurred (subjects 4, 9, 10, 11, 14, 15, 17, 26, 19, 20, 21, 22, 24, 25, 27, 28, and 30) when the superposed stimuli produced an illusion stronger than the single O-K figure ($I_S > I_{O-K}$) and six cases (2, 3, 8, 12, 13, and 23) when the superposed stimulus illusion exceeded the M-L effect ($I_S > I_{M-L}$). Certainly, the summation coefficient r_S varied among subjects: e.g., the difference between the psychophysical result (41 arc min) and the arithmetical sum (47 arc min) was 6 arc min (two steps) and r_S was 0.87 for subject 28, but the difference $I_{O-K} + I_{M-L} - I_S$ was 27 arc min (nine steps) and r_S was 0.64 for subject 30.

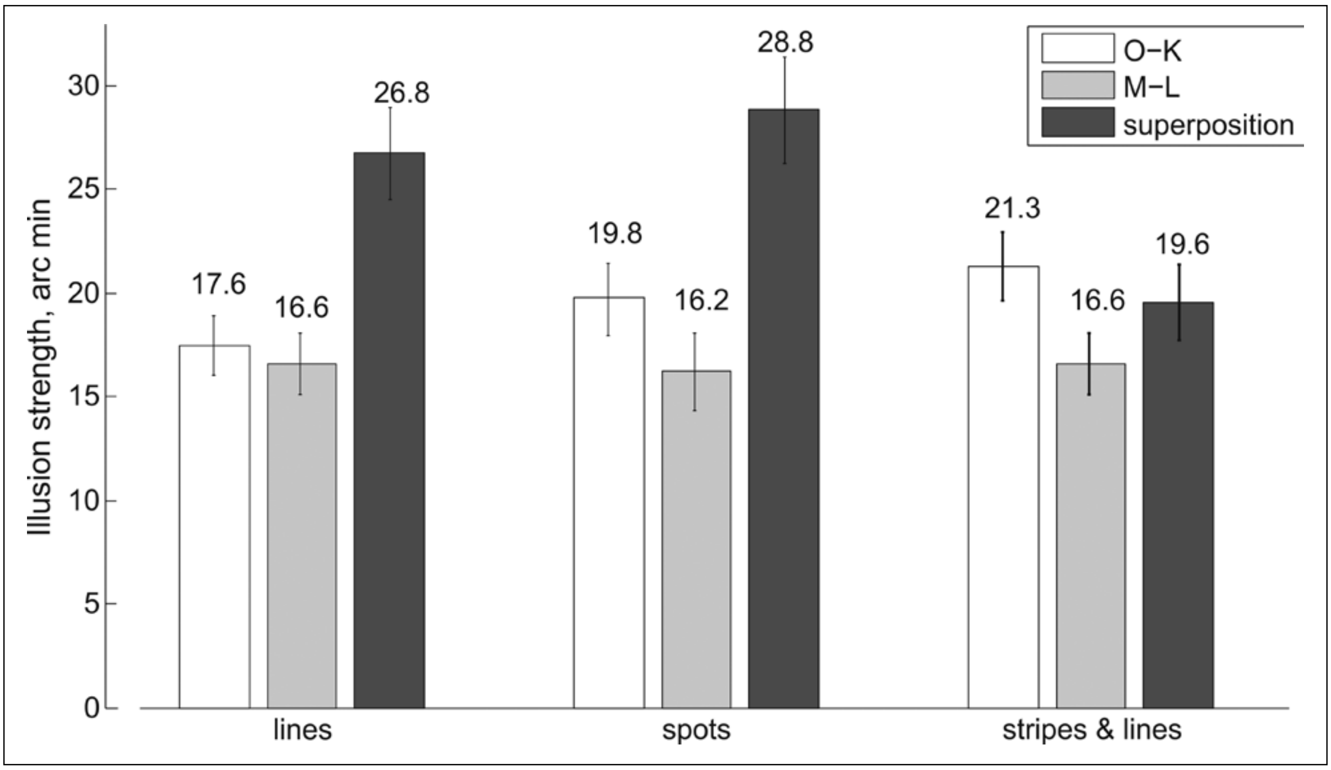


Fig. 5. Averaged magnitudes of the perceived distortions: the Oppel-Kundt, Müller-Lyer and superposed stimuli illusions correspond the white, gray and black bars. The first bar set presents the data collected by the stimuli formed of lines; the second set gives those of spots; and the third one shows the results of the vertical stripes in the O-K filled interval with the flanking Müller-Lyer wings formed of lines. In all stimuli, the length of the M-L wings is 20 arc min. Error bars, 0.95 confidence intervals. Data for 30 subjects. See main text for a detailed explanation.

The over-summation illusion as reinforcement was detected in the data of Fig. 3*A*. The superposed stimuli yielded a larger illusion value than the total of the two separate illusions (subjects 5 and 6). Summation absence was not observed in the data for all thirty subjects.

The mean value of the illusion given by the superposed figures formed of lines was 26.8 arc min (30%; *SD*=13.6; Fig. 5). The compound illusion was significantly greater than O-K (t_{29}=5.57, P<0.05) and greater than M-L illusion (t_{29}=6.19, P<0.05).

If the superposed stimuli were formed of spots (Fig. 3*B*), the increased illusion effect also appeared in the data for all subjects. The complete summation was seen in data from three subjects (1, 3, and 7) and incomplete summation appeared in 23 cases: fifteen (4, 9, 10, 12, 17, 26, 19, 20, 22, 24, 25, 27, 28, 29, and 30) with IS > IO-K and eight cases (2, 8, 13, 14, 15, 18, 21, and 23) with IS > IM-L. Four subjects (5, 6, 11, and 16) illustrated over-summation events. Two of them (5 and 6) showed the effect repeatedly (Fig. 3*A*).

The mean values of the illusion given by the superposed figures formed of spots was 28.8 arc min (32%; *SD*=14.9; Fig. 4). This was greater than O-K (t_{29}=5.50, *P*<0.05) and greater than M-L (t_{29}=7.61, *P*<0.05).

When the vertical stripes' O-K stimulus was combined with the M-L wings formed of lines, the increase in the combined illusion strength was less evident (Fig. 3*C*) than that seen in the experiments with the line and spot stimuli (Fig. 3*A* and Fig. 3*B*). Only 14 observers showed the incomplete summation event: ten (3, 5, 6, 13, 15, 17, 23, 27, 28, and 30) with $I_{M\text{-}L}<I_S<I_{O\text{-}K}$ and four (2, 4, 11, and 26) with $I_{O\text{-}K}<I_S<I_{M\text{-}L}$. No complete summation or over-summation cases appeared in this experiment. Sixteen subjects (1, 7, 8, 9, 10, 11, 12, 14, 16, 18, 19, 21, 22, 24, 25, and 29) demonstrated the summation absence. One subject (9) may have concentrated on the O-K figure while manipulating the superposed stimulus because $I_S \approx I_{O\text{-}K}<I_{M\text{-}L}$. Six subjects (8, 12, 14, 18, 21, and 25) preferred the M-L wings as $I_S \approx I_{M\text{-}L}<I_{O\text{-}K}$. Nine subjects (1, 7, 10, 11, 16, 19, 22, 24, and 29) strayed between the M-L wings and the O-K filling-stripes because their results were: $I_S<I_{O\text{-}K}<I_{M\text{-}L}$ (10); $I_S<I_{M\text{-}L}<I_{O\text{-}K}$ (7 and 16); $I_{M\text{-}L}<I_S<I_{O\text{-}K}$ (1, 11, 19, 24, and 29); and $I_S \approx I_{O\text{-}K} \approx I_{M\text{-}L}$ (22). Fifteen of sixteen subjects had values of r_S<0.5, and one had r_S=0.5 (Table III).

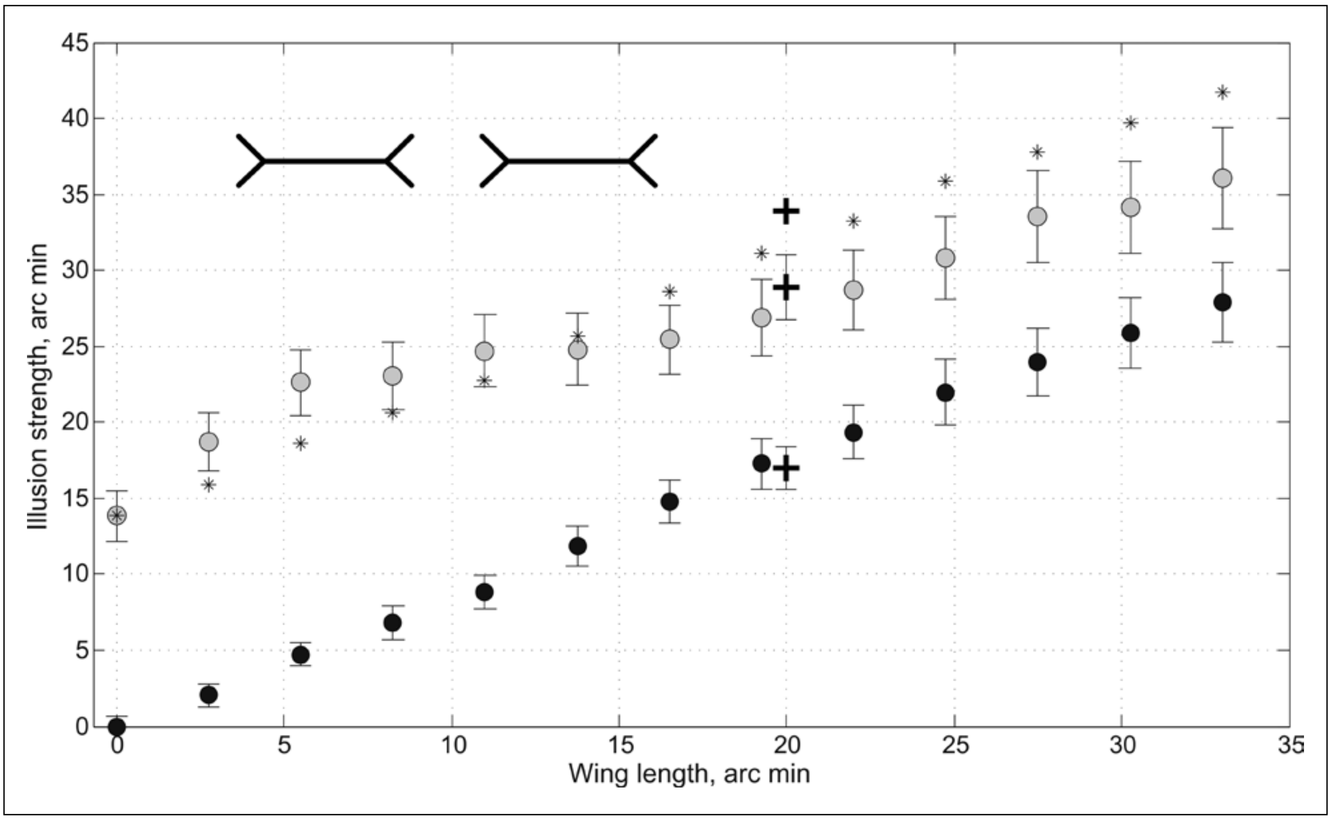


Fig. 6. Compound illusion strength at the Oppel-Kundt and Müller-Lyer stimuli superposition (*C3*) as a function of the Müller-Lyer wing length, gray circles. The Müller-Lyer illusion strength, dark circles. The Oppel-Kundt illusion value added to the Müller-Lyer values, stars, to show the arithmetical sum of two illusions measured separately. The averaged data for 29 subjects. Three crosses in the position of the 20 arc min of the wing length represent the data from experiment 1 (Fig. 5, first bar set). Error bars indicate 0.95 confidence intervals.

The mean value of the illusion given by the superposed figures formed of vertical stripes and wing lines was 19.6 arc min (22% of the reference distance; SD=12.2; Fig. 5). This was below the mean value for the O-K illusion

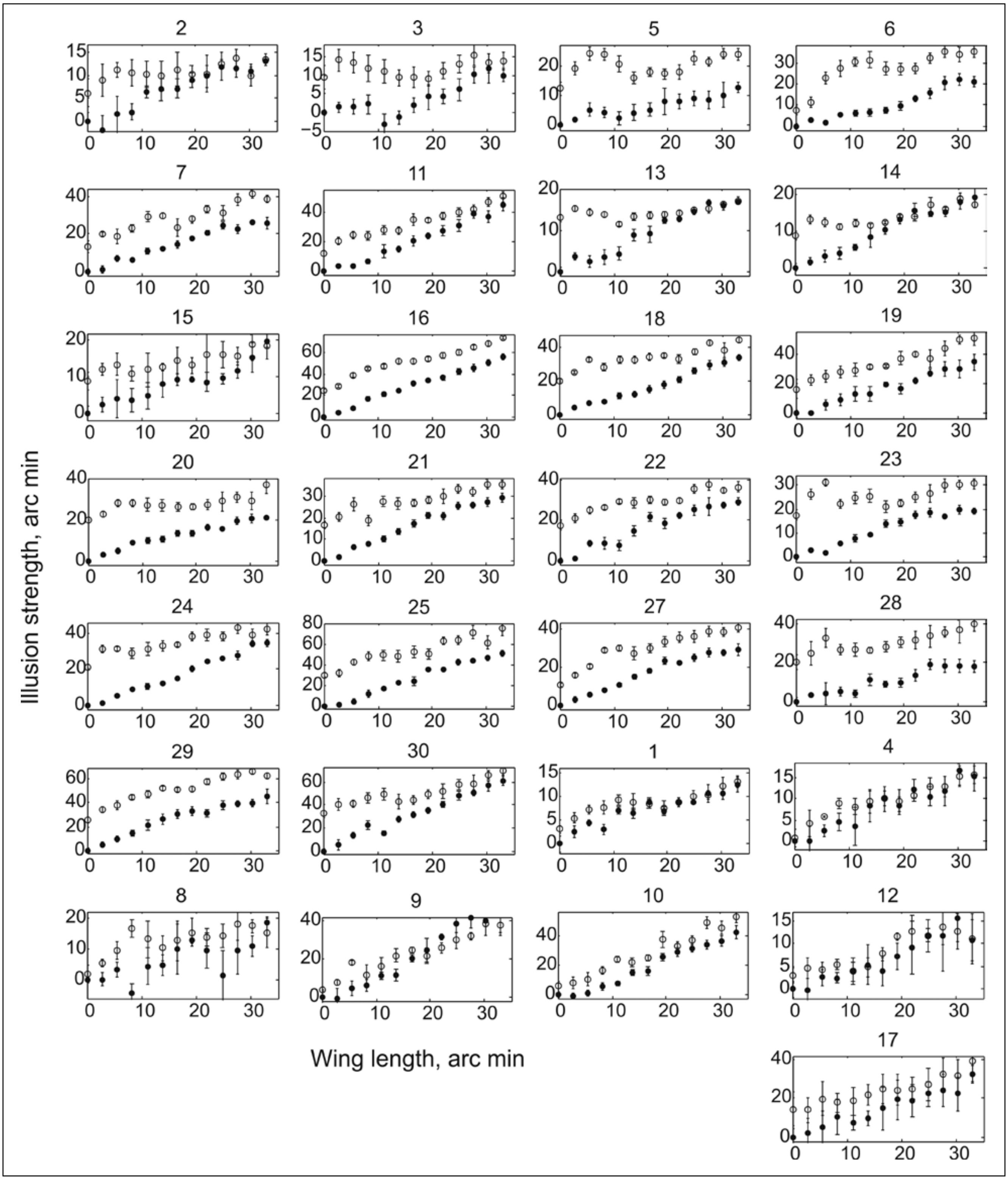


Fig. 7. Individual subject data (the frame numbers) on the compound illusions strength in dependence on the M-L wing length, light circles. The M-L illusion strength, dark circles. Error bars, 0.95 confidence intervals.

Table III. Variation in the length comparison errors according to data from Figs. 2 and 3. The 0.95 confidence intervals were estimated using the χ^2 method.

A. Mean of STDs

Stimulus	Lines	Spots	Vertical stripes
O-K	4.2 (3.3–5.6)	4.6 (3.6–6.2)	4.5 (3.6–6.1)
M-L	4.3 (3.5–5.8)	4.1(3.2–5.5)	
Superposition	4.8 (3.9–6.5)	5.4 (4.3–7.3)	4.5 (3.6–6.0)
Control		2.9 (2.3–3.9)	

B. STD of the means

Stimulus	Lines	Spots	Vertical stripes
O-K	9.5 (7.6–12.7)	10.5 (8.3–14.1)	10.4 (8.3–14.0)
M-L	9.0 (7.2–12.2)	10.4 (8.3–14.0)	
Superposition	13.6 (10.8–18.3)	14.9 (11.9–20.1)	12.2 (9.7–16.3)
Control		4.6 (3.6–6.1)	

(21.3 arc min) but above the mean value for the M-L illusion (16.6 arc min). Related to that the O-K stimulus with vertical stripes and attached M-L wings was less inducible than the stimuli formed of horizontal line segments or spots. The summation coefficient values emphasized the reduced influence effect. The O-K illusion for lines (first column in Fig. 5) reached the 17.6 arc min value, the M-L illusion took 16.6 arc min, and the superposed stimuli produced the 26.8 arc min illusion, i.e., the sum of parts (34.2 arc min) was larger than the whole by 7.4 arc min. The summation coefficient was 0.78 (r_S=26.8/(16.6+17.6)=0.7836). For the stimuli of spots (second column set in Fig. 5), the sum of parts (19.8 arc min+16.2 arc min=36.0 arc min) was larger than the whole (28.8 arc min) by 6.9 arc min. Again, the illusion summation with a coefficient of about 0.8 appeared true (r_S=28.8/36.0=0.8). In contrast, the illusion magnitude at superposition of the vertical stripes and wings of line segments (19.6 arc min; third column set in Fig. 5) was below the O-K magnitude (21.3 arc min). Coefficient r_S approached 0.52 (r_S=19.6/21.3+16.6=0.517), indicating that the O-K mechanism signals were relatively weak in this particular experiment with this particular stimulus.

The data obtained in Experiment 1 illustrated the compound illusions dependence on individual properties of visual signal processing and spatial structure of the stimuli exposed. Dynamics of summation in dependence of the M-L wings' length was represented in Experiment 2.

Experiment 2

Experimental data for 29 subjects (Fig. 6) showed a regular increase in the length misperception magnitude (gray circles) with elongation of the M-L wings from 0 to 34 arc min in superposed stimulus C3. On average, the compound illusion strength was greater by 12.2 arc min (t_{12}=12.3, P<0.05) than the M-L illusion (black circles). The arithmetical sum of the M-L and O-K illusions (stars) paralleled the M-L results because the O-K illusion strength (13.9 arc min) was constant in all presentations of the superposed stimuli (and was measured directly by the stimulus in which the M-L wings were reduced to zero). As it shown in Fig. 6, the experimental and arithmetical summation data were similar within the 2.8–14 arc min interval of wing length, indicating complete summation (at the 11 and 13.8 arc min wing lengths) and the effect of over-summation (at 2.8, 5.5, and 8.3 arc min of wing length). The summation coefficient decreased for the wings longer than 14 arc min, but within the 23 – 34 arc min interval, it remained stable between the 0.85 and 0.9 limits.

The averaged results of the illusion strength measured in experiment 1 with single and superposed stimuli (Fig. 5) essentially fit the data obtained in experiment 2 (Fig. 6).

The curves in Fig. 7 showed individual variations in the summation results. The summation was easily seen in the data of 22 subjects (2, 3, 5, 6, 7, 11, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, and 30). The summation was less pronounced but present for the other subjects (1, 4, 8, 9, 10, 12, and 17). Over-summation appeared for subjects 2, 5, 6, 18, 23, 24, and 27 in cases with short wings. Noticeably, subjects 5 and 6 demonstrated the effect again.

DISCUSSION

The main target in the present study was the neurophysiological event, summation of magnitudes of two illusions of extent, rather than concepts on the illusions origin. The experimental task was to measure: (a) the M-L and O-K illusions manifestation and (b) overestimate of the stimulus interval flanked by arrowheads facing outward (M-L) but having smooth or regular filling (O-K), when compared to underestimate of an empty stimulus interval with inward M-L wings. Nonetheless, our experimental findings account slightly for reasons of the distance estimation biases in the filled/unfilled space stimuli and may address the views on the phenomenon genesis. Early theories (Hering 1861, Kundt 1863, Wundt 1898) and 20th century approaches (Bulatov et al. 1997, Craven 1993, Craven and Watt 1989, Lewis 1912, Spiegel 1937, Tausch 1954, Watt 1990) have not provided a complete understanding of misperception. Effectively, the O-K phenomenon has been recognized as a multi-factorial event since it depended on the number, form, size, orientation, spatial density, luminance contrast, and polarity of the filling elements (Wackermann and Kastner 2009, 2010, Wackermann 2012b). The O-K illusion per se involved a number of components: the space expansion due to the low level spatial filtering (Bulatov et al. 1997, Bulatov and Bertulis 1999, 2005); expansion because the spatiotemporal integration along the continuous excitation path (Bertulis et al. 2014); spatial anisotropy of the expansion effect (Wackerman et al. 2012a); biases of the perceptual localization of the stimulus terminals because the integrated context-evoked neural excitation (Bulatov et al. 2017); and terminal repulsion (Mikellidou 2012, Mikellidou and Thompson 2014), the size of which is insufficient to account for the total illusion magnitude, but tolerable to fit the magnitude difference in the uniform and discrete filling with the optimal number of the contextual elements. Though the computational modelling (Bulatov et al. 1997, 2017, Erdfelderand and Faul 1994, Wackermann and Kastner 2010) provided adequate description of certain illusory effects, the underlying neural mechanisms and their localization in the brain remained speculative. Both subcortical structures, such as colliculus superior, and cortical areas related to the contour, shape, and attention expressions might become desirable candidates.

The present experimental findings support evidently the need of the multivariate approach to the O-K phenomenon. Our data showed extensive variability in the O-K illusion magnitude (and in the M-L illusion as well) from relatively small (2–3% of the reference distance) to quite large (43–50%). A wide norm of the visual reaction to the O-K and M-L stimuli during the length comparison procedure indicated that the size estimate system integrated signals of numerous neural events during the experiments. The output of the proper mechanisms and procedures, e.g., low-level spatial filtering, positioning the gravity center, spatial-temporal integration of excitation along the real or illusory contours, or spatial anisotropy of the expansion effect could be corrected by the higher-level assessment of spatial relationships of the stimulus elements (Coren and Girgus 1972, Predebon 1998, Weidner and Fink 2007) and influenced by attendant factors, such as gaze fixation and attentional pooling (Bulatov et al. 2009), duration of observations (Bertulis et al. 2014), learning and training (Judd 1902, Parker and Newbigging 1963), motivation and attitude (Bates 1923), age (Pollack 1970), education, culture, and visual ecology (Davis 1970). The attendant factors rather than the illusions' primary inducers caused a variability in the experimental data. Indeed, the variation in illusion strength was about twice smaller for a subject's individual data (Table II*A*) than for several subjects' data (Table II*B*).

The length comparison results were still more variable at the stimuli superposition. The standard deviation of the means exceeded the mean of the standard deviation by almost three times (Table II). The control stimuli were the simplest patterns used in the experiments and caused the least deviation (Table II). Taken together, the present data illustrated certain rigidity in the proper illusory mechanisms and flexibility in the individual experimental accomplishment.

The combined illusion strength was greater than that of a single stimulus. Summation of the two effects was regular and reproducible in the data for all 30 subjects in Experiment 1 and in all 29 subjects in Experiment 2. The mean values of illusion strength measured in Experiment 1 and Experiment 2 did not differ. One may recognize the addition of perceived distortions as a property of vision, even though summation appearances varied among subjects, and the summation coefficient covered the scale from 0.5 to 1. Furthermore, four subjects had summation coefficients > 1, indicating that the superposed stimuli yielded larger illusion value than that of two separate illusions put together. Although an explanation of the over-summation event can hardly be suggested at present, and more detailed studies of the effect are needed, it did not contradict the data on addition of the two distortions of extent.

The mean summation coefficient was 0.8 for the stimuli of spots and 0.78 for those of lines.

Concerning the character of summation of the two illusions of extent, one seeks to answer, why the summation effect did not occur (the summation coefficient was<0.5) when relatively high filling stripes of the O-K stimulus were combined with the M-L wings formed of line segments. A possible qualitative explanation may be related to the unity perception hypothesis (Bertulis et al. 2014), according to which a spatiotemporal integration of excitations along the real or illusory contours of the filled space gives rise to a distorted percept. One may suppose that just the high stripes (66 arc min) in stimulus *D3* made the O-K component less influential because the subjects were instructed to match the distances between the apexes of the M-L wings. The positions of the wing needles forced the subjects' eye to operate along the horizontal stimulus axis when estimating the length of the intervals in the superposed stimuli. Consequently, the upper and lower ends of the vertical stripes of the O-K component might be off the attentional pooling, the size of which is about 3–5 min of arc at the fovea center and about 25–40 min of arc in 1° periphery (Intriligator and Cavanagh 2001, Nakayama and Mackaben 1989, Sagi and Julesz 1986). Thus, the illusory contours of the filling body could be hardly perceived during the length matching procedure, and the spatiotemporal integration processes could not be acting. As a result, the O-K component in the combined response became small if any. But if some subjects possessed broader attentional pooling, e.g., due to eye movements, the O-K contribution was present, for instance, 14 of 30 subjects showed the effect of incomplete summation in the experiment. In contrast, if a subject concentrated on the contours of the stripe filling in the experiment with stimulus *D3*, the M-L effect might weaken, and the combined illusion might become about equal in strength to O-K alone (Table III and Fig. 3*C*). Finally, if a subject strayed between the length judgments criteria (stripe ends continuity and apexes of the M-L wings) the combined result did not reach the magnitude of a single illusion.

For stimuli *B3* and *C3*, the regular spot sequences or horizontal line segments of the O-K component were laid just on the stimulus axis like the apexes of the M-L wings. Accordingly, both illusions manifested themselves in the combined response without any exception.

The O-K and M-L illusions appeared to be two separate components in response to the superposed stimuli indeed. The combined illusion strength varied in a regular way in dependence on a single spatial parameter of one of the components, in particular, on the M-L wing length. The short wings, such as 2.8–14 arc min, provided the complete summation pattern. The over-summation result was suspected for stimuli with 2.8, 5.5, and 8.3 arc min wings. The long wings of 22.8–34 arc min caused the summation coefficient to decrease to 0.85–0.9. Individual subject data (Fig. 7) varied but provided no contradiction against the summing-up approach.

Alternative explanations for the present data on the illusions summation can be hardly suggested by the involvement of such factors like anisotropy of the expansion effect, direction-specific edge detectors or biases of the perceptual localization of stimulus terminators.

In general, the present experimental data indicated that the O-K and M-L illusions were self-dependent events that developed in the neural networks when the appropriate stimuli appeared superposed and presented simultaneously. The two misperceptions arose and combined into a single sensory output depending on the spatial parameters of the stimuli and individual properties of the visual signal processing.

CONCLUSIONS

Experimental testing of subjects naive to the goals of the study revealed extensive variability in the magnitudes of the two best known geometrical illusions on perceived extent: 3–43% of the reference distance in the M-L stimulus (formed of spots or line segments) and 2–50% in the O-K figure (formed of spots, line segments, or vertical stripes) with the average strength 18% (M-L) and 22% (O-K).

The superposed M-L and O-K patterns caused evidently stronger distortions of perceived extent than the same stimuli presented separately: 32% for the stimuli formed of spots and 30%, for the stimuli of line segments.

The O-K pattern of vertical stripes in the superposed stimulus was less inducible, and the compound illusion appeared relatively weak, 22% in the average strength.

The summation coefficient for the superposed stimuli formed of spots was 0.8, for the stimuli of lines 0.78, and for the stimuli of vertical stripes 0.52.

The summation coefficient varied in dependence on the length of the M-L wings. For relatively short wings formed of line segments (3-14 arc min), the coefficient value approached 1.0. For longer segments, the value decreased.

The experimental findings supported: a) an assumption that the M-L and O-K misperceptions may combined into sensory signal with proportions depending on the spatial parameters of the superposed stimuli and on the individual properties of the visual processing and b) an explanation of the O-K illusion in terms of continuity perception and spatiotemporal integration along a continuous excitation path.

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