

# Complexity analysis of spontaneous EEG

---

**Joydeep Bhattacharya**

Commission for Scientific Visualization, Austrian Academy of Sciences,  
Sonnenfelsgasse 19/2, A-1010 Vienna, Austria, Email: [joydeep@oeaw.ac.at](mailto:joydeep@oeaw.ac.at)

---

**Abstract.** The aim of the present paper is the assessment of the overall complexity of spontaneous and non-paroxysmal EEG signals obtained from three groups of human subjects, e.g., healthy, seizure and mania. Linear complexity measure suitable for multi-variate signals, along with nonlinear measures such as approximate entropy (ApEn) and Taken's estimator are considered. The degree of linear complexity is significantly reduced for the pathological groups compared with healthy group. The nonlinear measures of complexity are significantly decreased in the seizure group for most of the electrodes, whereas a distinct discrimination between the maniac and healthy groups based on these nonlinear measures is not evident.

---

**Key words:** EEG, complexity, approximate entropy, Taken's estimator, noise, seizure, mania

The Electroencephalogram (EEG) is an objective noninvasive measure of the dynamical activity of the brain that provides not only a local but also a global spatiotemporal description of the collective neuronal activity. In spite of its wide application in assessment of the functioning of the brain, meaningful analysis of EEG signal still remains a challenging task because of the complex irregular nature of the signal, which can be nonstationary as well as nonlinear. Traditionally, EEG is characterized as a realization of a linear stochastic process. With the backing of strong developments in understanding the statistical behavior of random process, this idea brought numerous applications in clinical field and they were used as a "gold-standard" over decades (Niedermeyer and Lopes da Silva 1993). In relatively recent years there has been increasing interest in the nonlinear dynamical analysis of the EEG signal for an improved understanding of brain functions. It was proposed that at least in some modes, EEG is generated by a low-dimensional chaotic neuronal process (Babloyantz and Destexhe 1986, Soong and Stuart 1989). It is now well recognized (Kantz and Schreiber 1997) that direct and blind applications of nonlinear system theory are often misleading and quite unjustified in mathematical terms (Glass and Mackey 1988). Also the popular question of "Is it noise or chaos?" is almost improper to ask in the analysis of EEG since both the order and disorder are embedded in the activity of neuronal assemblies (Freeman 2000). In this paper, a very modest and straightforward question is posed. Given two sets of EEG signals, emanating from two subjects in different clinical states, can one discriminate between them? Obviously there will be an infinite number of possible ways to look at the EEG data, but the main concern of this paper is to compare the EEGs through assessment of the degree of complexity hidden in the complex and aperiodic signal. Three measures of complexity are used here, namely the linear complexity, the dimensional complexity and the approximate entropy (ApEn).

The background EEG signals were recorded from electrode locations Fp1, Fp2, F7, F3, F4, F8, T3, C3, C4, T4, T5, P3, P4, T6, O1, O2 on the scalp according to the standard 10-20 International electrode placement system. The electrodes are numbered (Fig. 1) from 1 to 16. An average reference was used. The subjects were placed in a sound proof, light attenuated air-conditioned (20°C) room and instructed to relax and close their eyes for some time during the data acquisition period. The sampling frequency was 200 Hz and the signal was fil-

tered between 0.1-70 Hz. A notch filter of 50 Hz was also used. The data were stored on an optical disk drive and then later transported to a hard disk for further analysis. Twenty four subjects were chosen belonging to three broad groups (each group consisted of eight subjects): control subjects with no reported psychiatric or neurological disorders, subjects with maniac symptoms, and subjects with seizure. The subjects were within the age range 18-65 years with the mean ages of 32.50, 30.44, and 28.44 years for the three groups respectively. All the subjects gave written consent prior to the recording. An epoch length of ~10 s of uninterrupted EEG data, which were free from any visual complexes, (e.g., the spike wave complexes for seizure patients) were chosen for analysis. Baseline drift was removed by subtracting a polynomial of 2nd order.

Three linear descriptors were computed from the multichannel signal: a measure of global field strength ( $\Sigma$ ), a measure of global frequency of field changes ( $\Phi$ ), and a measure of spatial complexity ( $\Omega$ ). The first two measures are related to Hjorth's complexity (Hjorth 1973) originally proposed for single channel EEG. This "omega complexity" ( $\Omega$ ) was earlier introduced in the

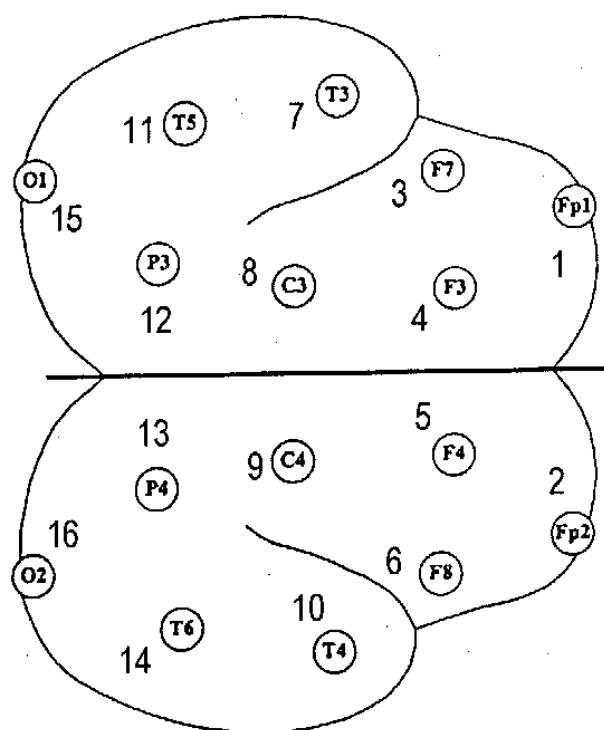


Fig. 1. Positions of the 16 electrodes including their number and their designations.

analysis of covariance matrix formed by multichannel data as spatial principal component analysis (Morgera 1985, Palus et al. 1991), but these three measures together were popularised by Wackerman (1996, 1999) and a variant of these was applied to multichannel EEG data (Klonowski et al. 1999). Sixteen channel data ( $\{x_1(k)\}, \{x_2(k)\}, \dots, \{x_{16}(k)\}$ ) are stacked column wise to form a multichannel EEG matrix of dimension  $L \times 16$ , where  $L$  is the length of signal (Wackermann 1996). Typically speaking, each row ( $u_n$ ) of the matrix corresponds to the state vector representing the spatial distribution of EEG amplitudes over the entire scalp at that instant of time. Further, the signal is centered to zero mean value in all channels ( $\sum_n u_n = 0$ ) and is also transformed to the average reference ( $\sum_n u_n^i = 0$  for each one) (Wackermann 1999). The first two descriptors are computed as follows

$$\Sigma = \frac{1}{L} \sqrt{\frac{\sum_n \|u_n\|_2^2}{16L}}$$

$$\Phi = \frac{1}{2\pi} \sqrt{\frac{\sum_n \|u_n\|_2^2}{\sum_n \|(u_n - u_{n-1})/\Delta t\|_2^2}}$$

where  $\Delta t$  is the sampling period,  $u_n$  is the row vector of the multichannel EEG matrix, and  $\|\cdot\|_2$  is the 2-norm.

The covariant matrix (dimension  $16 \times 16$ ) of the above multichannel EEG matrix is formed as

$$C = \frac{1}{L} \sum_n u_n u_n^T$$

Next the eigenvalues  $\{\lambda_1, \lambda_2, \dots, \lambda_{16}\}$  (Golub and Van Loan 1996) of the matrix  $C$  are obtained and subsequently normalized (normalized eigenvalue,  $\xi_i = \lambda_i / \sum_i \lambda_i$ ). The quantity,  $\Omega$  is computed as follows

$$\log \Omega = - \sum_{i=1}^{16} \xi_i \log \xi_i$$

In terms of topographic mapping,  $\Sigma^2$  is the mean squared global field power and  $\Phi$  is the rotation rate of the hypothetical circular trajectory in the phase space. Omega ( $\Omega$ ) roughly quantifies the amount of spatial synchronization. Large values of  $\Omega$  indicate no linear spatial correlation between different electrodes whereas the low values correspond to the minimal complexity or high correlation. Earlier, whole night sleep EEG have been successfully characterized in this parametric space of

three linear complexity measures (Szelenberger et al. 1996). In this paper, these three descriptors were calculated for non-overlapping data windows of 2.5 s and then averaged and subsequently compared between different groups.

The most widely used parameter in nonlinear dynamics is the correlation dimension introduced by Grassberger and Procaccia (1983). Despite its countless application in EEG analysis, one may obtain "spurious" evidence of low-dimensional chaos if the embedding parameters are not chosen properly (Kantz and Schreiber 1997). An alternative but statistically more informative, Taken's maximum likelihood estimator (Takens 1985) of the correlation dimension is used here. Each channel is treated individually. In the phase space reconstruction procedure, the time lag is set to the first local minimum of the mutual information, and the Theiler's window to exclude temporally correlated points is set to the first zero crossing of the autocorrelation tim. This estimator is computed as

$$d_T(r) = \frac{C(r)}{\int_0^r \frac{C(r')}{r'} dr'}$$

where  $C(r)$  is the conventional correlation integral, and  $r$  is the radius of hyperspheres in the reconstructed phase space. It has to be stressed that this nonlinear descriptor is not used with the aim of indicating low dimensional chaos, since in no cases are strict scaling criterion found; rather it is used as an operational and effective measure (Lehnertz and Elger 1998).

Another measure of complexity, called approximate entropy (ApEn) was proposed by Pincus (1991), which was successfully applied to relatively short and noisy data (Pincus 1991, Pincus and Goldberger 1994). In EEG analysis, there are very few reported results (Diambra et al. 1999) of the application of ApEn. Two parameters  $m$  and  $r$  must be chosen prior to the computation of ApEn where,  $m$  specifies the pattern length, and  $r$  is the effective filter. Here also, one has to compute the correlation integral  $C^m(r)$  (with embedding dimension  $m$  and time lag 1). This measure is finally obtained as follows

$$ApEn(m, r, L) = \frac{1}{L - m} \sum_{i=1}^{L-m} \log C_i^{m+1}(r) - \frac{1}{L - m + 1} \sum_{i=1}^{L-m+1} \log C_i^m(r)$$

Thus ApEn quantifies the (log)likelihood that sets of patterns that are close remain close on next incremental comparison. Smaller values of ApEn imply a greater likelihood that certain patterns of measurements will be followed by similar measurements. If the time series is highly irregular, the occurrence of similar patterns in the future is less likely. For this study,  $m$  is set to 2 and  $r$  is set to 15% of the standard deviation of each time series. These values are selected on the basis of previous studies indicating good statistical validity for ApEn within these variable ranges (Pincus and Goldberger 1994). It should be mentioned that ApEn is not an approximation to the standard Kolmogorov-Sinai entropy rather it is a regula-

tory statistic, which can be used for the comparison between different time series.

The EEG time series recorded by the O2 electrode are shown in Fig. 2a-c for three subjects belonging to three different groups. It is clear that all the signals represent the spontaneous background activity free from paroxysmal complexes. An analysis of variance (ANOVA) is performed to compare the complexity outcome of the pathological groups with the control group. The probability of  $P < 0.05$  is set as the significant level.

Table I depicts the statistical outcome of the linear complexity analysis between the control group and the two pathological groups. It is found that the measures  $\Sigma$  and  $\Phi$  are significantly lower ( $P < 0.01$ ) in the seizure group than the control group but the maniac subjects show no significant change from controls. The omega complexity is found to be lowest in the seizure group (mean = 4.16, SD = 0.09,  $P < 0.001$ ); the healthy group possesses the highest complexity (mean = 5.12, SD = 0.05) whereas the complexity of the maniac group lies in somewhere between (mean = 4.46, SD = 0.35,  $P < 0.01$ ). The increased complexity suggests that there are more independent, parallel, functional processes active in the control group than the pathological group.

Figure 3a-f show the mean profile of Taken's dimension estimator and ApEn for the three groups, computed for each channel separately. It is clearly evident from simple visual inspection that the seizure group possesses the lowest nonlinear complexity. Table II lists the  $P$  values found in ANOVA studies for comparison of  $d_T$  and ApEn. Out of the 16 channels, only three channels (Fp1, C3, C4) fail to be significantly lower in the seizure group than the control group with  $P > 0.05$ . So the loss of nonlinear complexity is found to be almost globally valid for seizure subjects. But in the maniac group, the dimension values are not significantly different from the healthy

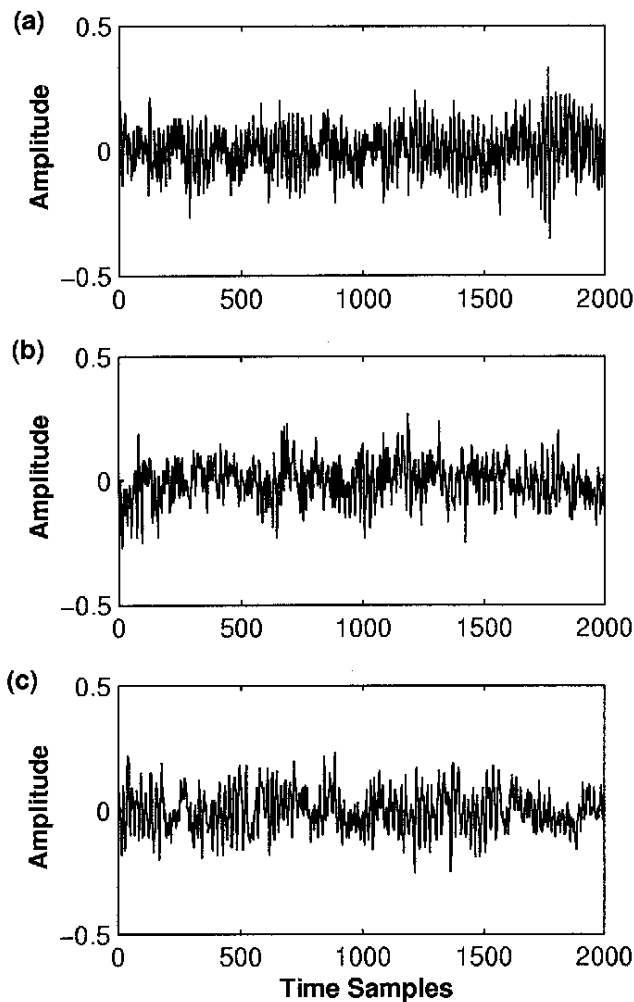


Fig. 2. a-c, time series of EEG signal of 10 s (= 2,000 time samples) duration, recorded at electrode O2 from three subjects belong to three broad groups namely healthy, seizure and mania respectively. All the time series look very irregular and aperiodic.

Table I

Statistical outcome of the linear complexity descriptors. Probability ( $P$ ) values are obtained through comparison with the control group

Linear Measures	$P$ values for Seizure group	$P$ values for Mania group
$\Sigma$	0.005	0.418
$\Phi$	0.015	0.515
$\Omega$	0.00009	0.010

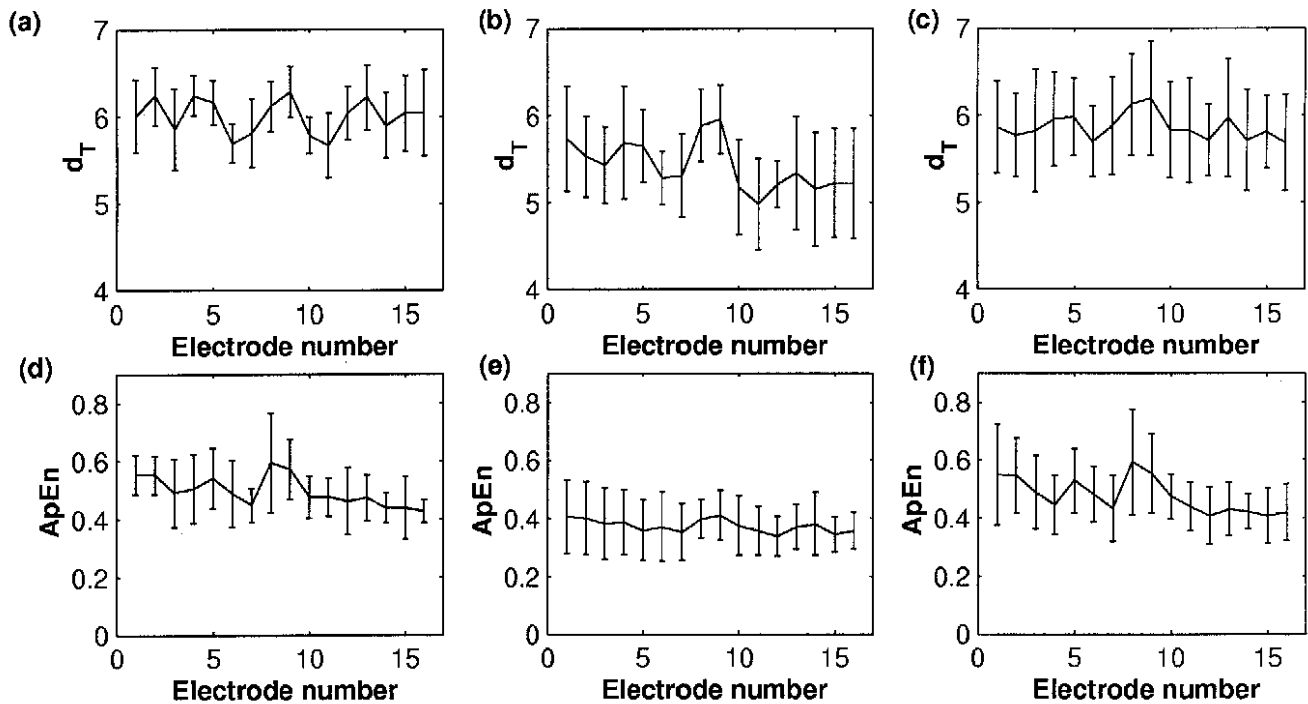


Fig. 3. a-c, mean value of Taken's estimator,  $d_T$ , ( $\pm$  SD) of 16 electrodes for three different groups (control, seizure and mania) respectively (See Fig. 1 for identification of the electrodes against their number). d-f, mean value of ApEn ( $\pm$  SD) of 16 electrodes for three different group respectively. The profiles of the mean values for healthy and mania group are strikingly similar.

Table II

Statistical outcome of the analysis for Taken's maximum likelihood estimator of dimension and approximate entropy. Probability ( $P$ ) values are obtained through comparison with the control group

Electrodes	$P$ values for Taken's estimator		$P$ values for ApEn	
	Seizure	Mania	Seizure	Mania
Fp1	0.003	0.042	0.009	0.895
Fp2	0.316	0.565	0.011	0.938
F8	0.009	0.961	0.060	0.904
F7	0.083	0.915	0.898	0.956
C4	0.011	0.809	0.034	0.942
C3	0.038	0.776	0.033	0.743
T6	0.014	0.474	0.180	0.577
T5	0.009	0.529	0.006	0.329
O2	0.011	0.195	0.017	0.834
O1	0.009	0.312	0.005	0.540
F4	0.010	0.342	0.033	0.826
F3	0.401	0.197	0.050	0.297
C4	0.081	0.731	0.004	0.766
C3	0.211	0.973	0.009	0.988
p4	0.005	0.385	0.017	0.327
p3	0.00004	0.094	0.021	0.335

group except two channels (Fp1, P3). It is interesting to note that the central region (C3, C4) and electrodes closer to the mid-line always possess high values, whereas the occipital region (O1, O2) always possesses low values for all the subjects irrespective of their clinical status.

The loss of spatio-temporal complexity in seizure patients is well reported (Weber et al. 1998) for the interictal phases, for EEG recorded intracranially, whereas in this research the result is corroborated through easily obtainable noninvasive EEG signals. It has been reported (Pezard et al. 1996) that depressive subjects also display a lower level of linear complexity than controls. For maniac subjects, nonlinear complexity analyses do not produce any significant differences from the control group whereas the linear measure does. Several comments are noteworthy in this respect. The sophisticated nonlinear measures are more prone to noise contamination in the data, but the linear measures based on Karhunen-Loeve Transform or Singular Value Decomposition can orthogonally segregate the noise subspace from the signal subspace. The measurement noise is very likely to disturb the inherent neuronal signal during recording of EEG; so this noise might influence the actual outcome of the nonlinear analysis. Further, the differentiation between noise and low dimensional chaos in a spatially extended system like the brain is very difficult (Grassberger 1989). The existence of high dimensionality restricts the successful attractor reconstruction from a finite length noisy signal. Although here the linear measure is found to be more useful to some extent, the utility of nonlinear analysis cannot be ruled out either. Finally we note that it will also be interesting to compute other complexity measures (Wackerbauer et al. 1994) to get a more comprehensive overview of the underlying process.

The author wishes to thank S.H. Nizamie for providing the data. He would also like to acknowledge E. Pereda, J. Wackermann, W.F.G. Mecklenbraeuer, P.P. Kanjilal, E. Wenger, and H. Petsche for various helps and suggestions. TISEAN software (<http://www.mpi-pks-dresden.mpg.de/~tisean>) (Hegger et al. 1999) was used for estimating the Taken's estimator.

Babloyantz A., Destexhe A. (1986) Low dimensional chaos in an instance of epilepsy. *Proc. Natl. Acad. Sci. USA* 83: 3513-3517.

Diambra L., Bastos de Figueireda J.C., Malta C.P. (1999) Epileptic activity recognition in EEG recording. *Physica A* 273: 495-505.

Freeman W.J. (2000) A proposed name for aperiodic brain activity: stochastic chaos. *Neural Networks* 13: 11-13.

Glass L., Mackey M.C. (1988) *Rhythms of life: from clocks to chaos*. Princeton University Press, Princeton, NJ, 248 p.

Golub G.H., Van Loan C.F. (1996) *The matrix computations*. Johns Hopkins University Press, Baltimore, MD, 694 p.

Grassberger P. (1989) Information content and predictability of lumped and distributed dynamical systems. *Physica Scripta* 40:346-353

Hegger R., Kantz H, Schreiber T. (1999) Practical implementation of nonlinear time series methods: The TISEAN package. *Chaos* 9: 413-435.

Hjorth B. (1973) The physical significance of the time domain descriptors in EEG analysis. *Electroencephalogr. Clin. Neurophysiol.* 34: 321-325.

Kantz H., Schreiber T. (1997) *Nonlinear time series analysis*. Cambridge University Press, Cambridge, 320 p.

Klonowski W., Jernajczyk W., Niedzielska K, Rydz A., Stepień R. (1999) Quantitative measure of complexity of EEG signal dynamics. *Acta. Neurobiol. Exp.* 59: 315-321.

Lehnertz K., Elger C. (1998) Can epileptic seizures be predicted – evidence from nonlinear time series analysis of brain electrical activity. *Phys. Rev. Lett.* 80: 5019-5022.

Morgera S.S. (1985) Information theoretic complexity and its relation to pattern recognition. *IEEE. Trans. Syst. Man. Cybernet.* 15: 608-619.

Niedermeyer E., Lopes da Silva F.H. (1993) *Electroencephalography: basic principles, clinical applications, and related fields*. Lippincott Williams & Wilkins, Baltimore, MD, 1258 p.

Palus M., Dvorak I., David I. (1991) Remarks on spatial and temporal dynamics of EEG. In: *Mathematical approaches to brain functioning diagnostics* (Eds. I. Dvorak and A.V. Holden). Manchester University Press, Manchester, p. 369-385

Pezard L. et al. (1996) Depression as a dynamical disease. *Biol. Psychiat.* 39: 991-999.

Pincus S.M. (1991) Approximate entropy as a measure of system complexity. *Proc. Natl. Acad. Sci. USA* 88: 2297-2301.

Pincus S.M., Goldberger, A.L. (1994) Physiological time-series analysis: what does regularity quantify? *Am. J. Physiol.* 266: H1643-H1656.

Soong A.C.K. Stuart C.I.J.M. (1989) Evidence of chaotic dynamics underlying the human alpha-rhythm electroencephalogram. *Biol. Cybernet.* 62: 55-62.

Szelenberger W., Wackermann J., Skalski M., Niemcewicz S., Drojewski J. (1996) Analysis of complexity of EEG during sleep. *Acta Neurobiol. Exp.* 56: 165-169.

Takens, F. (1985) On the numerical determination of the dimension of an attractor. In: *Dynamical systems and bifurcations* (Eds. B.L.J. Braaksma, H.W. Broer and F. Takens). *Lect. Notes in Math.* 1125, Springer, Heidelberg.

- Wackermann J. (1996) Beyond mapping: estimating complexity of multichannel EEG recordings. *Acta Neurobiol. Exp.* 56: 197-208.
- Wackermann J. (1999) Towards a quantitative characterization of functional states of the brain: from the non-linear methodology to the global linear description. *Int. J. Psychophysiol.* 34: 65-80.
- Wackerbauer R., Witt A., Atmanspacher H., Kurths H., Scheingraber H. (1994) A comparative classification of complexity measures. *Chaos Solit. Fract.* 4: 133-173.
- Weber B., Lehnertz K., Elger C.E., Wieser, H.G. (1998) Neuronal complexity loss in interictal EEG recorded with foreman ovale electrodes predicts side of primary epileptogenic area in temporal lobe epilepsy: a replication study. *Epilepsia* 39: 922-927.

*Received 7 February 2000, accepted 1 June 2000*