

## Deterministic brain oscillations in the magnetoencephalogram

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**Abstract.** Determinism is a special property of some systems and is defined by its state-space behavior in which the trajectories in time never intersect. Whether or not determinism exists in brain activities is a question that may be resolved by analysis of the dynamical properties of the electroencephalogram (EEG) or magnetoencephalogram (MEG). We will show that even though there are strong nonstationarities in most brain behaviors, small epochs of deterministic dynamics can still be observed. We will also show that the local Lyapunov exponents are measures that can demonstrate smooth transitions into these deterministic states.

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**Key words:** magnetoencephalography, nonlinear dynamics, time series, nonstationarity

## INTRODUCTION

Due to the strong nonlinear character of the brain dynamics (Babloyantz and Destexhe 1986, Rapp et al. 1990, Elbert et al. 1994, Muhlnickel et al. 1994, Pritchard et al. 1995, Kowalik et al. 1996) standard linear methods are frequently not applicable for the characterization of normal or pathologic brain states. This is because spectral properties of brain oscillations are very individual and may vary in time and space. Though brain signals (EEG/MEG) looks very irregular, it is possible that these are generated by deterministic and mathematically simple nonlinear systems. The number of variables required for the description of such a system corresponds to the dimensionality of the system's dynamics. The controversy about whether or not low-dimensional chaotic behavior (i.e., with relatively simple mathematical description) is present in the EEG (Babloyantz and Destexhe 1986, Rapp et al. 1990) arises because of important issues that are concerned with the preprocessing (filtering) of the data (Albano and Rapp 1993, Pritchard et al. 1995) and the sensitivity of dynamical measures to noise. Furthermore the EEG and MEG signals are not only nonlinear but they are also nonstationary, and therefore they do not fulfill the axiomatic requirements for using the standard dynamical measures. The question of the determinism in brain dynamics is not definitively resolved (Elbert et al. 1994, Muhlnickel et al. 1994) and needs to be studied more thoroughly.

Some authors reported stable values of dynamical measures estimated for EEG signals recorded from different groups of subjects or for different mental tasks (Lutzenberger et al. 1992). Tests for determinism (Kaplan and Glass 1992, Muhlnickel et al. 1994) (or for nonlinearity) support the hypothesis about non-noisy mechanisms of cortical oscillations. Nevertheless, many investigators doubt the validity of a low-dimensional character of the EEG signal (Frank et al. 1990, Theiler 1995, Theiler and Rapp 1996). Both positions may be true, but only if we suppose that the deterministic brain oscillations suddenly appear in a nonstationary manner, with a transitory low-dimensional character. In this case it should be possible to observe rapid transitions between the hypothesized low- and a high-dimensional states. The experimental results presented below support the hypothesis about the transient deterministic states of the brain. The results seem to suggest that brain dynamics consist of a series of different oscillatory processes that corre-

spond to specific neuronal activities. In an uncontrolled case (i.e., spontaneous EEG/MEG activity), the oscillations, even if low-dimensional, appear randomly and thus form an unpredictable system. We found among electrodes distributed widely over the head that there were some locations that manifested the repetition of, at least one, low-dimensional chaotic state (possibly being coupled to some uncontrolled cognitive process).

## METHODS

### Methods and experimental setup

Using whole-head magnetoencephalography (Neuro-mag-122) we measured cortical activity in 11 patients with focal epilepsy and four control persons. The results for the clinical outcome studies will be presented elsewhere (Kowalik et al., unpublished). This study focuses on the question of deterministic low-dimensional brain dynamics that we observed in all subjects.

The recordings (5 min eyes closed + 5 min eyes open, sampled at 259 Hz) of spontaneous activity (resting condition) were used to examine the spatial distributions of local Lyapunov Exponent (ILE). The eyes movement artifacts were fully controlled with simultaneous recordings of vertical and horizontal EOGs. All 122 channels of recorded EEG/MEG activity corresponding to 61 sensor locations (two-dimensional vector in the orthogonal cortex plane) were used in the presentation of the results.

Our hypothesis about transitory deterministic character of EEG/MEG does not allow the application of standard nonlinear methods because the main assumption is there that estimated time series are generated by a stationary system. For instance, the Lyapunov Exponent (LE) is able indeed to detect weak changes in the dynamical signal development (Kowalik et al. 1993, Kowalik and Elbert 1994, Kowalik et al. 1997), and it may, in some situations, distinguish between different macroscopic states of the neuronal activity (Kowalik et al. 1996). Unfortunately, in a nonstationary case this measure does not support stable values when the dynamics changes within the time window required for reconstruction of the high-dimensional attractor. The local Lyapunov Exponent (ILE), on the other hand, may describe the same property as would occur with a stationary epoch of data, but it is designed to be able to characterize the brief momentary dynamics of transient deterministic

behaviors (Wolff 1992, Kowalik et al. 1996). Time dependent local LE is defined as:

$$\lambda_{i,m} = \frac{1}{m} \frac{1}{n_i} \sum_{j \in S_i} \log \left| \frac{X_{i+m} - X_{j+m}}{X_i - X_j} \right|$$

where  $i = 1, \dots, n-m$  with  $n$  being the length of the time series (or the width of the running window) whereas  $m$  denotes the time-lag;  $S_i = \{j: \varepsilon < |X_i - X_j| \leq V\}$ ,  $n_i = \#(S_i)$ ;  $m \in \mathbb{N}$  and  $V > \varepsilon$ .

In order to estimate successive values  $\lambda_i$  for an arbitrary chosen value  $m$  one needs to find  $n_i$  sets of indices  $S_i$  for which the locality condition is fulfilled, i.e., distances between a running point and other points are in the range  $(\varepsilon, V)$ . We introduced a parameter  $\varepsilon$  (noise limit) in order to eliminate zero-distances that cannot exist in real, i.e., noisy time series. These zero-distances occurring during the computation can drastically change a final value of  $\lambda$ . The  $\lambda$ -time-series pattern depends in addition on the locality parameter ( $V$ ) and on the assumed level of noise ( $\varepsilon$ ). Nevertheless, we found with the help of simulated signals (below) that it is possible to localize frequency-unspecific dynamical changes of MEG. The estimation of specific value in the  $\lambda$ -time-series depends also on the width of the applied window. The choice of this parameter is important because the quasistationary behavior within it will be assumed. This condition establishes the upper limit of the window-width. This is an experimental fact that in the spontaneous activity data it is in the range 0.5-4 s (Rapp et al. 1990, Elbert et al. 1994, Kowalik and Elbert 1994). The lower limit of this range is coupled to the time scale of observed processes and with the ILE-method itself. Unfortunately, there is no simple rule how to find its correct value. Let us observe the role of the time scale on a real signal as shown in Fig. 1.

The estimation of ILE was performed there for one channel of the MEG data. Three values of the width of running window were applied: 64, 512, and 4096 points which correspond to 250 ms, 2 s, and 16 s respectively. The time interval presented in Fig. 1 contains the transition between states: eyes closed-eyes open that takes place at  $t = 300$  s. For a window  $\Delta t = 250$  ms (64 points) the value of ILE is sometimes negative and there is no clear differentiation between „normal“ dynamics and a significant dynamical change of cortical dynamics between these two phases. Also in the case of  $\Delta t = 16$  s (4096 points) the behavior on both sides is not distinctly

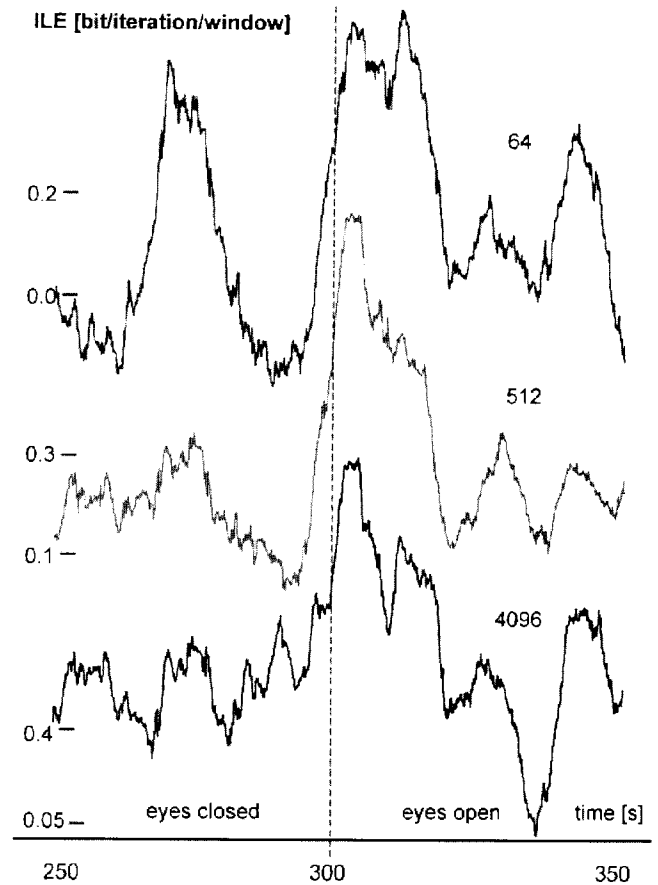


Fig. 1. Local LE estimated for one (occipital) MEG channel measured in a subject at resting condition (5 min eyes closed/5 min eyes open). Three values of running window were applied: 64 points ( $\approx 250$  ms), 512 points ( $\approx 2$  s), and 4096 points ( $\approx 16$  s). In all cases the same averaging window of 128 points ( $\approx 500$  ms) was applied. The dashed line surrounds the presented ILE pattern into two physiologically different states.

differentiable. The middle pattern obtained for  $\Delta t = 2$  s (512 points) fulfills our intuitive expectation - the critical change at  $t = 300$  s is clearly represented and all ILE values are there positive.

### Data analyses

The time series were examined with the ILE ( $t$ ) using a running window of 512 points ( $\approx 2$  s). A vicinity parameter  $V \approx 2.5\%$ , of a normalized range of amplitudes  $[0 \dots 4095]$ , noise limit  $\varepsilon = 10$ , and the vicinity  $V = 100$ . Only the first 5 min were examined to exclude contamination by eye blink artifacts that invariably occurred within longer recording epochs.

## RESULTS

### Simulations

To demonstrate sensitivity of the ILE as a detector of distributed dynamical changes, we performed simulations with artificial signals. For this purpose, all 122 channels which correspond to 61 geometrical positions in the head layout were substituted by Gaussian noise generators, and then additional components were added into the signal. In six left-central channels the signal generated by a function modeling a chaotic pendulum ( $\ddot{x} + a\dot{x} + c\sin(x) = \Lambda\sin(2\pi ft)$ ,  $a = 0.5$ ,  $c = 1.0$ ,  $\Lambda = 1.5$ ,  $f = 0.106$ ) were the data. The data epochs of the remaining 116 channels were randomized-phase surrogate data (Rapp et al. 1994, Theiler 1995, Pritchard and Stam 2000) prepared from epochs of the same chaotic-pendulum data. The results of this simulation are presented in Fig. 2.

It should be emphasized that neither the frequency spectrum nor the autocorrelation function can distinguish such site specific dynamics (Theiler 1995, Casdagli et al. 1997) but that the ILE can. This result is presented in Fig. 2A. The boxes inside the figure show FFT-spectra of the surrogate data in narrow (0-5 Hz) and wide (0-50 Hz) frequency bands. The ILE of surrogate data lie above the original ones. It is easy to see the fall of ILE(t)-values corresponding to the chaotic data channels (Fig. 2B).

### Experimental evidence of transitory low-dimensional dynamics

When looking at ILE(t) series for experimental MEG-signals one can see a number of different changes in brain dynamics; these sometimes correspond to visible changes in the raw signal. Dynamical changes in ILE that occur without significant changes in the spectral properties cannot be detected by linear methods. One such

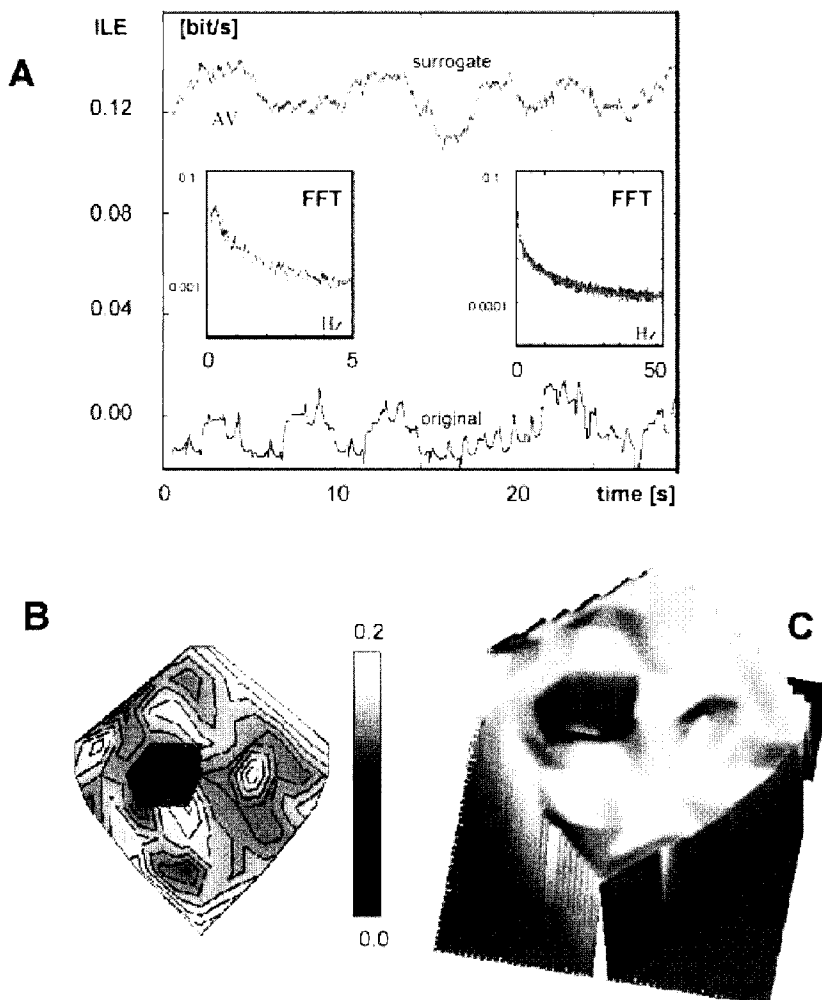


Fig. 2. Local LE applied for simulated pendulum data (A). FFT, boxes show spectral identity of original and surrogate data sets. The line AV denotes averaged position for 10 successive randomization performed for surrogate data. 2D- (B) and 3D (C) presentation of the ILE topography of synthesized activity applied to 122-channels MEG system. 6 left-central channels contain pendulum chaos data and gaussian noise (20% in amplitude) whereas into other 116 channels the surrogate data (and noise) were injected.

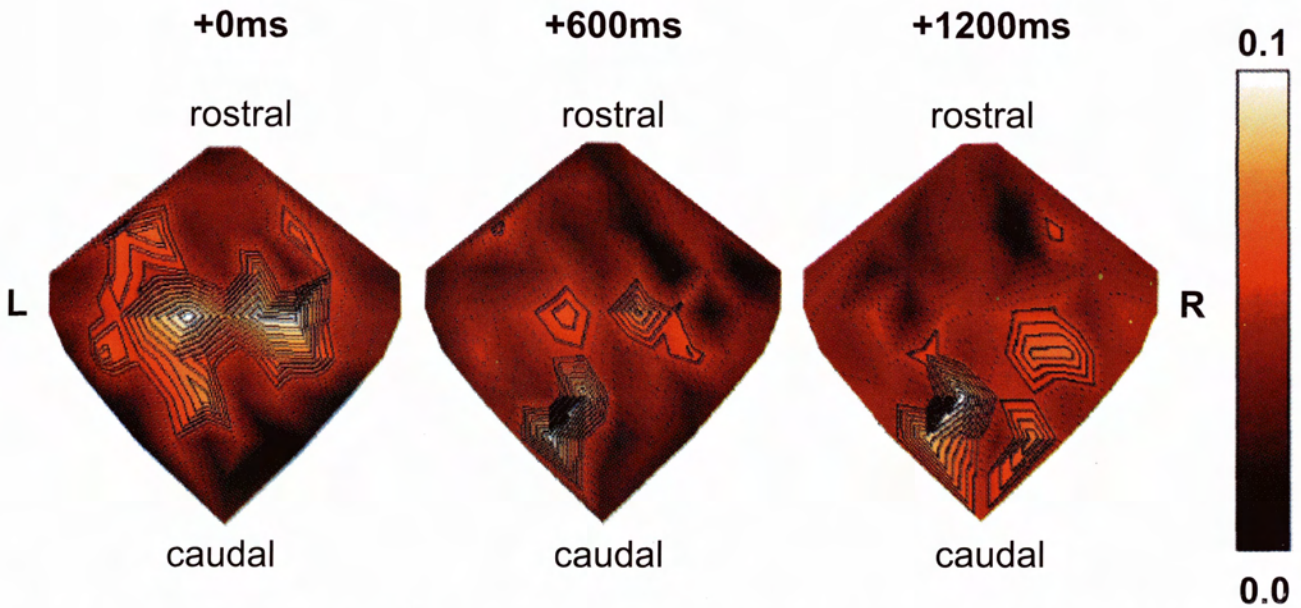


Fig. 3. Temporal change (1.2 s interval, step 600 ms) of 2D-spatial distribution of ILE of the MEG activity measured in one patient with focal epilepsy (left temporal lobe).

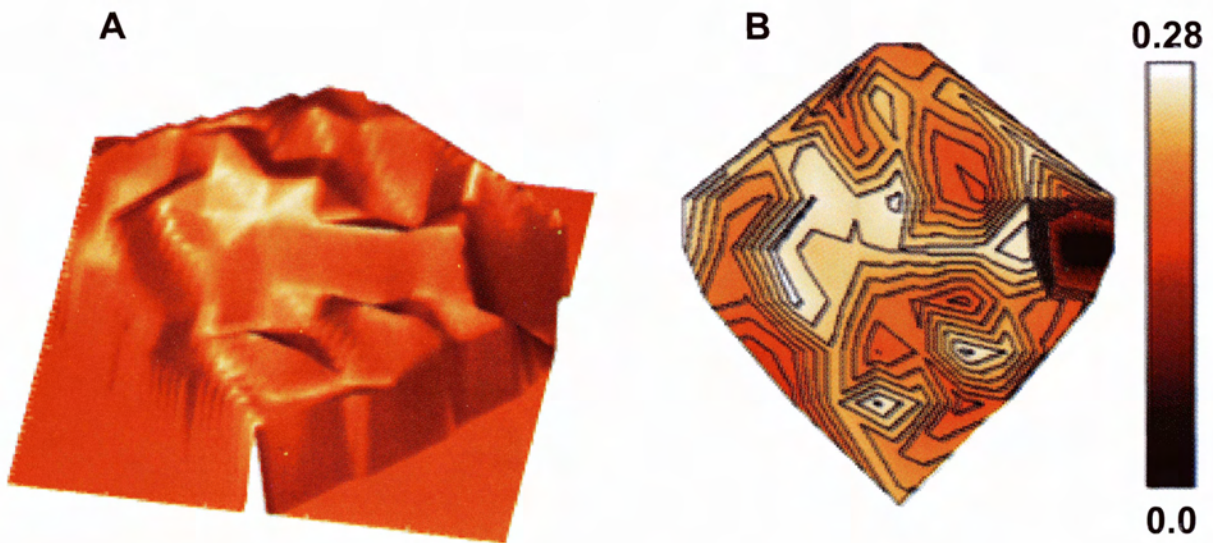


Fig. 5. Spatial 2D- (A) and 3D- (B) representation of the mean variability of ILE (standard deviation) in one patient with focal epilepsy (left temporal) estimated for 5 min (eyes closed) MEG-data. The mountains characterize these positions where the signal is mostly nonstationary as frequently changes of dynamics occur.

example is presented in Fig. 3. The 2D-contour plot shows the results over the whole scalp and demonstrates 3 snapshots of ILE values taken with 2 s running win-

dow<sup>1</sup>. The left sensor sites are stable over 15 s, and the right sites remained unchanged for the following 20 s. The spatial transitions occurred in the left-central region

<sup>1</sup>An animated version of figures presented in this paper will be available under <http://www.neurologie.uni-dusseldorf.de/meglab/zjk/non-noisy.html>.

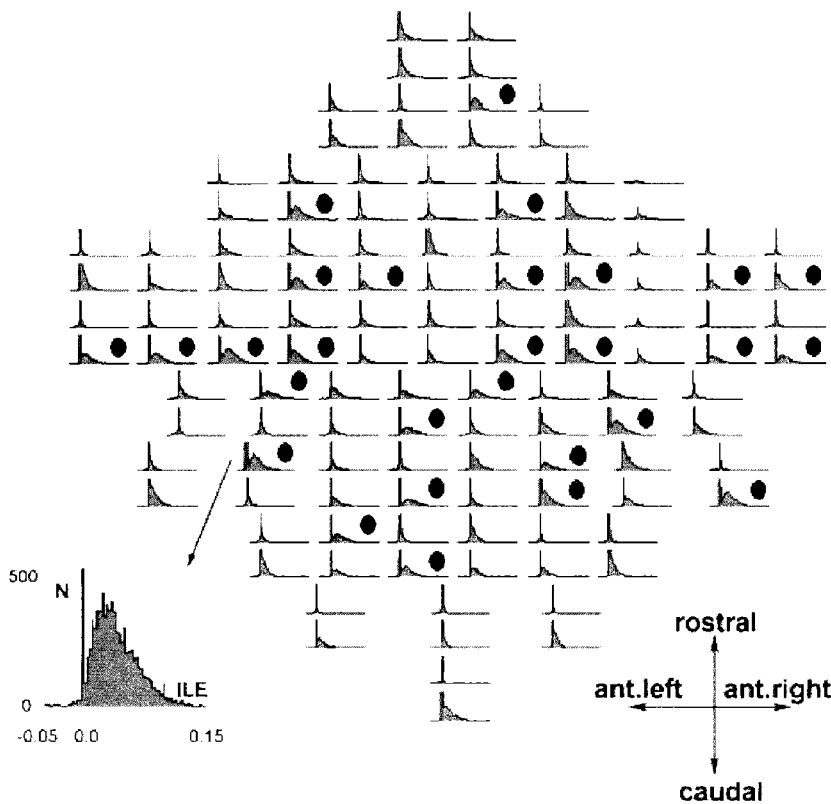


Fig. 4. 122 histograms for frequency of appearance of local LE values constructed for patient with right mesiotemporal-lobe epilepsy. There are 61 positions with two channels/location measured. Channels labeled with a spot exhibit low-dimensional chaotic behavior visible here as a distinct local positive maximum. The distributions around zero value are coupled with ubiquitous line frequency (50 Hz). The raster for the dynamical change, i.e., the width of diagram window is fixed to 128 boxes with a begin at minimal observed ILE-value, and an end at its largest value.

corresponding to the large change of the average ILE-values.

Since the ILE is a function of time, it reveals changes in the cortical activity that are also time-dependent. What then is the most probable time-dependent dynamical state of these cortical generators? A statistical description of the transition dynamics is presented in Fig. 4, which shows a histogram of the momentary ILE values appearing in the considered ILE (t) time series.

This diagram can be read as a „spectrum of dynamics”: negative values correspond to harmonic oscillations, zero - to quasiperiodic, and positive to a complex behavior, respectively. The greater is the ILE the higher dimensional dynamics occurs in the system. Many times repeated dynamics causes then an appearance of a peak in this spectrum. Note, there is no distinct limit between high-dimensional and noisy behavior. The locations of interest are therefore those with a small positive ILE values (local maxima on the positive side of the ILE distribution). At these positions (spots in Fig. 4) the deter-

ministic<sup>2</sup> dynamics appear frequently enough to make visible differences in the histograms. This behavior is not restricted in space but is distributed over the whole surface of the brain. ILE-appearance histograms like that presented in Fig. 4 were estimated for all measured subjects. In all cases we obtained similar results - though individual distribution there were always many channels exhibiting low-dimensional dynamics characterized by non-zero peak in the ILE-spectrum.

An averaged map of the ILE values of the cortical surface activities recorded simultaneously outside the unstable region from the same subject is shown in Fig. 5.

The variability of the signal at each electrode is quite large. When the variability exceeds that expected by chance alone, it is said to be nonstationary. Note that the variability of the ILE values is largest in channels lying closest to the epileptogenic region (symptomatogenic zone). The application of linear analytic methods to the same EEG and MEG data did not reveal any significant differences in dynamics between any of the electrode

<sup>2</sup>We found saturated slopes in the  $\log C(R) - \log(R)$  plots in the filtered data (5-13 Hz) in some channels. Also the measure for determinism (Kaplan and Glass 1992, Mühlnickel et al. 1994) has values which are significantly larger than those for noise ( $0.4 > 0.2$ ).

sites. Such changes might have been noted in linear signal analysis of single-unit spike-activity, but that was not recorded.

The ordered spatio-temporal structure appearing in a "snapshot" of low-dimensional dynamics in the presented data describes not only a pathologic property in the tissue, but it also reveals a systematic change in the underlying process. In case of epilepsy it could be explained if one assumes that the neural cells belonging to the local lesion "enslave" the healthy cells (undertake the control) for a brief interval and then induce a new order in the global structure of the cortical activity. That is, a local induction of properties leads to the spreading of similar dynamical properties to neighboring channels. The evidence for this spreading is seen in the lighter region in left-central part of the 3D-plot of ILE-variability (Fig. 5).

## DISCUSSION

In our studies on epilepsy (this report and Kowalik et al., unpublished) we have supposed that the measured signal has at least two dynamical states, one corresponding to the semi-normal behavior that is high-dimensional, and another (low-dimensional) that is coupled to pathological synchrony of neuronal firing. The reduction of the dimensionality in the EEG data for epileptic patients has already been reported by other authors (Lehnartz and Elger 1995, Pijn et al. 1997). It should be stressed that such transitions between states characterized by different dynamical descriptors (ILE, D2, etc.) were observed here also, but were observed in normal subjects as well. We conclude that these nonlinear descriptors which do not require data stationarity reveal important quantitative characteristics of nonstationary brain behavior. Furthermore the nonstationarity itself may differentiate between normal and pathologic brain activities, and constitute an important discriminator of activity that cannot be made by those linear and nonlinear descriptors that require data stationarity.

An implication of our findings is that the random choice of some interval from a nonstationary data set can result in very different values for a given dynamical descriptor. For some sampled intervals, the attractor structure may be "less noisy" (i.e., the correlation dimension or Lyapunov exponent indicates a low-dimensional, highly deterministic dynamics) whereas for other samples there may be no saturation of the dimension or a very different rate of divergence. This observed within-sub-

ject nonstationarity may explain why some authors did or did not find a low-dimensional behavior in EEG signals (Babloyantz and Destexhe 1986, Frank et al. 1990, Pijn et al. 1997). The characterization of nonstationary dynamical changes (hyperdynamics) can be performed either by counting the number of transitions like we did previously (i.e., in data from patients with schizophrenia (Rockstroh et al. 1997) or by directly plotting the standard deviation of the averaged local Lyapunov exponents (ILE) as was presented above.

Whether or not the determinism of EEG/MEG signals presented in this study can be generalized to the global determinism of the brain activity is not fully clear. In order to answer this question, further investigations are necessary.

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