Visual interpolation of surfaces defined through motion

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Abstract. The ability of human visual system to interpolate surfaces when the structure of the objects was defined through motion was investigated in a series of experiments. The type of the surface, the position of an area devoid of dots (the gap) and the type of gap edges was varied. The local orientation of the interpolated surfaces was estimated by a method of adjustment. The results show that the interpolated surfaces possessed less depth than the simulated ones. The type of the surface and the position of the gap had a significant effect on the performance. The worst results were obtained for a surface with orientation discontinuity (dihedral angle) where not only the shape of the interpolated surface differed significantly from the simulated one but the variability of the estimates was largest as well. The type of the gap edges did not affect the performance. The results are discussed in relation to the algorithms of surface interpolation.

Key words: structure from motion, surface interpolation, slant and tilt, interpolation algorithm
INTRODUCTION

The relative movement of image elements due to self-motion or object-motion has long been recognized as one of the richest cues for the recovery of the three-dimensional structure of objects and their spatial relations. In this process the dynamic two-dimensional projections of image elements are not perceived as separate elements, but as elements lying on the surface, or inside the volume of the object. The interpolation of surfaces is a process in which depth values are assigned to areas devoid of features and a complete surface is reconstructed. A model of surface reconstruction in structure-from-motion displays has recently been proposed (Hildreth et al. 1995). It is suggested that the reconstructed surface is obtained by temporal smoothing a surface passing through the local depth values estimated from local velocity information. The theoretical considerations of Koenderink (1990), however, suggested that the visual system could bypass the extraction of local depth information. If this is the case, the interpolated surface could be obtained from other surface descriptors like local surface orientation or curvature instead of local depth values.

The empirical studies of visual surface interpolation from motion information are scarce. Saidpour et al. (1992, 1994) used surface interpolation paradigm for a cylindrical surface or for a surface with orientation discontinuity (dihedral angle). The subjects’ task was to adjust the amplitude of a probe moving in phase with the two-dimensional motion of the image elements. Thus the perceived local depth information was estimated. The results show that the type of the surface influenced subjects’ performance. For a smooth curved surface the probe was placed slightly outside the surface while for a surface with orientation discontinuity the probe was placed inside the surface as if the discontinuity was smoothed. In these experiments, however, the local depth of the interpolated surfaces was measured only over a plane curve.

The purpose of the present experiment was to provide quantitative data on the interpolation of different surfaces based on local orientation measures in structure-from-motion displays. The effect of surface type, position of the gap and the type of the gap edges on the interpolated surfaces was studied in order to establish the requirements and constraints to the algorithms for surface interpolation.

METHODS

Stimuli

Pseudo-random dot patterns were used as stimuli. A hexagonal grid was used to determine the dots coordinates in the two-dimensional projection of the surfaces in a position straight ahead. The dots in the grid were randomly shifted in any direction with a shift less than half the side of the hexagonal grid. The generation procedure ensured a minimal effect of texture density. Three types of surfaces were simulated: elliptic, hyperbolic and dihedral angle. For the elliptic surface the radii of curvature in the principal directions of curvature were 80 pixels in x- and z-directions and 100 pixels in y-direction. For the hyperbolic surface all radii of curvature were equal to 80 pixels. The dihedral angle was oriented vertically.

Fig. 1. Schematic representation of the surfaces used in the experiment.
and had size 70 deg. The mean number of visible dots inside the aperture was 150. A schematic representation of the surfaces used in the experiment is presented in Fig. 1.

A rectangular area devoid of dots (a gap) was positioned around the direction of maximal curvature. For the dihedral angle this is the direction perpendicular to the direction of the orientation discontinuity. The size of the gap was 40 pixels. Two different positions of the gap were used: symmetrical or asymmetrical with respect to the direction of maximal curvature. The asymmetrically positioned gap was shifted to the left or to the right by a displacement equal to half the size of the gap (see Fig. 2).

The edges of the gap were sharp (as if an external rectangular object occluded the surfaces) or smooth.

The stimuli were viewed through a circular aperture with radius of 70 pixels.

**Procedure**

The subjects sat in a darkened room at a distance of 64 cm in front of a computer screen. At this distance one pixel of the screen subtended 0.2 deg of arc. The subjects were presented with moving dot patterns. The two-dimensional projections of the movement corresponded to a rotation of an elliptic, or a hyperbolic surface or a dihedral angle around a vertical axis. An orthographic projection was used. The stimuli oscillated at $\pm 20$ deg with an angular velocity of 30 deg of arc/s. A gauge figure similar to that used by Koenderink, van Doorn and Kappers (1992) was presented at different positions in the gap. The figure moved in phase with the stimulus dots with amplitude determined by its position, i.e. as if it is rigidly attached to the surface (see Fig. 3). The subject’s
TABLE I

Experimental conditions used in the study

<table>
<thead>
<tr>
<th>Experimental session</th>
<th>Type of surface</th>
<th>Type of edges</th>
<th>Edge position</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. elliptic</td>
<td>sharp</td>
<td>symmetrical</td>
<td></td>
</tr>
<tr>
<td>2. hyperbolic</td>
<td>sharp</td>
<td>symmetrical</td>
<td></td>
</tr>
<tr>
<td>3. dihedral angle</td>
<td>sharp</td>
<td>symmetrical</td>
<td></td>
</tr>
<tr>
<td>4. elliptic</td>
<td>smooth</td>
<td>symmetrical</td>
<td></td>
</tr>
<tr>
<td>5. elliptic</td>
<td>sharp</td>
<td>asymmetrical</td>
<td></td>
</tr>
<tr>
<td>Control: horizontal gap</td>
<td>elliptic</td>
<td>symmetrical</td>
<td></td>
</tr>
</tbody>
</table>

The task was to adjust the orientation of the gauge figure so that it appeared as lying on the surface. The adjustment resulted in a change of the two-dimensional projections of the gauge figure. The orientation of the gauge figure was manipulated by the use of computer keys. When the subject was satisfied with her performance she had to press another key from the keyboard and the next presentation started.

Two female subjects NB and SN aged 40-42 years took part in the experiment. Both have normal or corrected to normal vision. Subject SN was unfamiliar with the purpose of the experiments. They participated in five experimental sessions (see Table I for the experimental conditions used in the experiments). In each session only one combination of experimental conditions was used. The gauge figure was presented at random nodes of an imaginary rectangular grid 7 x 15 placed inside the gap. The asymmetrically positioned gap was shifted to the left of the direction of maximal curvature for one of the subjects, while for the other the shift was to the right. In addition Subject NB participated in a control experiment with horizontally oriented gap in an elliptic surface. Each experimental session lasted for about 2 h. The experimental sessions were performed on different days.

RESULTS

The experimental data consisted of the tilt and slant values of the gauge figure at all positions of the imaginary rectangular grid 7 x 15 inside the gap. Using numeric methods (singular value decomposition) a least-squares solution was obtained for the depth values of the grid points based on the local orientation estimates. Examples of the reconstructed surfaces are presented in Fig. 4.

The surface type influenced the subjects' performance. The reconstructed surfaces obtained in sessions 1, 2, and 3 differed significantly from each other ($\chi^2$-squared values 308.67, 130.48 and 116.02 for NB for the comparison elliptic-dihedral angle, elliptic-hyperbolic, dihedral angle-hyperbolic surface and 233.6, 178.82 and 505.74 for SN respectively, critical value = 81.47). The subjects' performance was worst for the dihedral angle - not only the appearance of the reconstructed surface differed from the original one, but also the average squared error was greater (see Table II).

The type of the surface edges: smooth or sharp (experimental sessions 1 and 4) had insignificant effect on the performance ($\chi^2$-squared value = 18.76 for NB and 24.38 for SN, critical value = 81.47). The position of the gap with respect to the direction of maximal curvature had a significant effect on the reconstructed surface (sessions 1 and 5). When the gap was positioned asymmetrically, the maximal curvature of the reconstructed surface was biased towards the direction of the gap shift (see Fig. 5).

For both subjects the reconstructed surfaces contained less depth than the original ones. For all smooth surfaces (elliptic and hyperbolic) the reconstructed surfaces look more cylindrical along the direction parallel to the rotation axis and less in perpendicular direction. One possible reason for this outcome might be an underestimation of the surface slant or a bias in the perceived curvature of the stimuli in direction parallel to the axis of rotation. If this was the case, then when the gap is oriented horizontally it might be expected to obtain more cylindrical surface in the direction of the smaller side of the gap and a veridical reconstruction of the surface along the longer side of the gap. This prediction was tested in a control experiment with subject NB. Figure 6 shows a comparison between the reconstructed surface obtained for a vertical and a horizontal gap in an elliptic surface. The $\chi^2$-squared test shows that the reconstructed
Fig. 4. Examples of the reconstructed surfaces inside the gap obtained in the different experimental conditions. A, for a symmetrically positioned gap with sharp edges inside an elliptic surface. B, for a symmetrically positioned gap with sharp edges inside an hyperbolic surface. C, for a symmetrically positioned gap with sharp edges inside a dihedral angle. D, for a symmetrically positioned gap with smooth edges inside an elliptic surface. E, for an asymmetrically positioned gap with sharp edges inside an elliptic surface. F, for a symmetrically positioned horizontal gap with sharp edges inside an elliptic surface. Subject NB. Similar results were obtained for subject SN. All surfaces are plotted at an equal scale.

surfaces could be regarded as statistically equivalent ($\chi^2$-squared value = 26.63, critical value = 81.47). For the elliptic surface used in the experiment the radius of curvature in the direction of the gap longer side was 100 pixels for the vertically oriented gap and 80 pixels - for the horizontal gap (3.3 and 2.67 deg of arc respectively). This difference was not obtained for the reconstructed surfaces. Thus, the cylindrical reconstructed surfaces obtained in the experiment were not due to the fact that the local slant was not correctly perceived. A more probable explanation for the cylindrical appearance of the interpolated surfaces could be the inability of the visual system to interpolate the surface relief over larger distances, as the estimates of the surface relief are local in nature.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Session 1 elliptic</th>
<th>Session 2 hyperbolic</th>
<th>Session 3 dihedral angle</th>
<th>Session 4 elliptic smooth edges</th>
<th>Session 5 elliptic asymmetric gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>39.09</td>
<td>131.09</td>
<td>343.82</td>
<td>25.88</td>
<td>189.46</td>
</tr>
<tr>
<td>SN</td>
<td>26.29</td>
<td>7.88</td>
<td>320.09</td>
<td>32.10</td>
<td>148.87</td>
</tr>
</tbody>
</table>
Interpolation algorithms

The present results were compared with the predictions of different algorithms for surface interpolation in order to test their applicability to the performance of the visual system. Saidpour et al. (1992, 1994) considered the algorithms of Ullman (1976) and Kellman and Shipley (1991) for plane curves and the algorithm of Grimson (1981) for surface interpolation. These algorithms minimize some distortion measures and imply global variation optimization to obtain the best of all possible descriptions of the surfaces. The algorithms for plane curves are not directly applicable for the surfaces used in our experiment. If, nevertheless, the surfaces are re-

Fig. 5. A side-view c: the two-dimensional projection of the reconstructed surface inside a gap in an elliptic surface. A, symmetrically positioned; B, shifted to the right.

Fig. 6. Reconstructed surfaces obtained for a horizontal and for vertical symmetrically positioned gap inside an elliptic surface.
garded as a stack of plane curves, these algorithms could be applied with some additional constraints. The algorithms of Ullman (1976) and Kellman and Shipley (1991), however, could not predict the perceived shift in the direction of maximal curvature for asymmetrically positioned gap.

Grimson’s algorithm minimizes a quadratic functional based on the depth values of discrete points and interpolates the smoothest surface. However, when the area devoid of dots is large in comparison to the surface patch, the interpolated surface depends on the initial values passed to the algorithm. The effect of the initial depth values of the gap on the performance of the algorithm is represented in Fig. 7 for an elliptic surface. It is clear that the interpolated surfaces differed in appearance from the surfaces reconstructed on the basis of the experimental results.

When the initial depth values inside the gap were zero the reconstructed surface obtained by Grimson’s algorithm had some characteristics similar to the surfaces reconstructed from the experimental data. It was more cylindrical along the longer side of the gap and the position of the gap (symmetrical or asymmetrical with respect to the direction of maximal curvature) affected the shape of the interpolated surface (see Fig. 8). The sign of the curvature (concavity/convexity) obtained by the algorithm in this case was opposite. It could be assumed that the sign of the curvature was undetermined under the orthographic projections of the stimuli. The existing experimental data (e.g. Dejkstra 1994) show that subjects had difficulties in detecting the sign of the curvature. Ambiguities between concavities and convexities and an arbitrary depth measurement are not uncommon result in the recovery of the three-dimensional structure of ob-
Fig. 8. A, the reconstructed surface by Grimson’s algorithm inside an elliptical patch when the initial depth values inside the gap are equal to 0. This figure represents the central part of Fig. 7A. B, the side view of the reconstructed surface for a symmetrical gap. C, the side view of the reconstructed surface for an asymmetrical gap.

In their model on surface reconstruction Hildreth et al. (1994) considered several possibilities for the initial depth values in the vicinity of objects boundaries in order to avoid flattening of the reconstructed surface as a result of the application of Grimson’s algorithm. They suggested either to use the depth values of the background plane or to force the derivative of the depth along the boundary to be high. Other possibility that these authors considered was to constrain the surface orientation along an object boundary to be perpendicular to the line of sight and to the two-dimensional projection of the boundary contour. The requirements to an algorithm for interpolation of a gap inside a surface, however, should be quite different, as they should conserve a possible continuity of the surface then the gap was due to occlusion from external object.

DISCUSSION

The results of the present experiment show that the surface relief in structure-from-motion displays could be reconstructed on the basis of local orientation measures. The results confirmed the data of Saidpour et al. (1992, 1994) that in the presence of an orientation discontinuity the visual system interpolates a smooth surface. As the type of the gap edges had insignificant effect on the performance it might be suggested that the visual system reconstructs the surfaces in structure-from-motion dis-
plays by similar mechanisms for external or internal boundaries, i.e. irrespective of the cause for the boundaries - occlusion, self-intersection or bounding contours.

The results also show that the reconstructed surfaces were more cylindrical than the simulated ones. In experiments testing the effect of surface shading on the perceived relief Haan et al. (1995) also found that the reconstructed surface based on local orientation measurements contained more locally cylindrical surface shapes than present on the surface. They suggested that this effect could be caused by those underestimation of surface slant. Our results contradict this explanation, as the orientation of the gap (horizontal or vertical) did not affect the reconstructed surfaces. A possible reason for the more cylindrical appearance of the reconstructed surfaces might be that the visual system could not interpolate the surface relief over larger distances, as the estimates of the surface relief are local in nature.

The data of the experiments show that algorithms for plane curves interpolation (Ullman 1976, Kellman and Shipley 1991) or for surface interpolation based on discrete points and global optimization procedures (Grimson 1981) do not predict well the results of the experiment without additional assumptions or constrains. It seems that the interpolation of the surfaces is performed within restricted spatio-temporal limits. It might be suggested that the interpolation is determined by significant geometrical features like the points of maximal curvature on the internal or external contours and is not based only on the local depth values. The magnitude of other descriptors of the optic field as suggested by Koenderink (1990) could also be used. Additional experiments are necessary to determine the limits of local information integration and the processes involved in surface interpolation.

**CONCLUSIONS**

The interpolation is an optimization process used by the visual system in different tasks not only to ensure a larger accuracy of performance, but also to correct to some extent the optic input and to achieve its continuity in space and time. The surface interpolation in recovery of the three-dimensional structure provided by the two-dimensional projections of moving objects is a necessary stage in visual information processing. The existing algorithms based on minimization of some variation measures could not fully characterize the reconstructed surfaces. It seems that other alternatives that do not seek a global variation minimization should be found. The algorithms used by the visual system for the interpolation of surfaces in structure-from-motion displays are not global in nature and are restricted in space and time. This allows the representation of the object shape with higher precision and preservation of its peculiarities. The spatio-temporal limits for surface interpolation and the type of local information used in these processes need further investigation.

Surface relief in structure-from-motion displays could be reconstructed on the basis of local orientation measures. The interpolated surfaces possessed less depth than the simulated ones and are more cylindrical in direction of the longer side of the gap. Thus the visual system could not interpolate the surface relief over large distances. Similar mechanisms are used for surface reconstruction irrespective of the type of surface boundaries – internal or external.

**REFERENCES**


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